Assignment 3 Solutions

1. $P(J \cup B) = P(J) + P(B) - P(J \cap B) = 0.55 + 0.40 - 0.27 = 0.68$

2. $P(J \cup G) = P(J) + P(G) - P(J \cap G)$

   $\Rightarrow \quad P(G) = P(J \cup G) - P(J) + P(J \cap G) = 0.64 - 0.55 + 0.11 = 0.20$

3. $P(B \cup G) = P(B) + P(G) - P(B \cap G)$

   $\Rightarrow \quad P(B \cap G) = P(B) + P(G) - P(B \cup G) = 0.40 + 0.20 - 0.50 = 0.10$

4. To answer this question, we draw the Venn diagram and find the probability of each outcome (each mutually exclusive part of the diagram). We start from the inside and work our way out:

   $P(\text{JBG}) = P(\text{J} \cap \text{B} \cap \text{G}) = 0.06$

   $P(\text{Jets and Bombers only}) = P(\text{JBG}^c) = P(\text{J} \cap \text{B}) - P(\text{J} \cap \text{B} \cap \text{G}) = 0.27 - 0.06 = 0.21$

   $P(\text{Jets and Goldeyes only}) = P(\text{JB}^c\text{G}) = P(\text{J} \cap \text{G}) - P(\text{J} \cap \text{B} \cap \text{G}) = 0.11 - 0.06 = 0.05$

   $P(\text{Bombers and Goldeyes only}) = P(\text{J}^c\text{BG}) = P(\text{B} \cap \text{G}) - P(\text{J} \cap \text{B} \cap \text{G}) = 0.10 - 0.06 = 0.04$

   $P(\text{Jets only}) = P(\text{JB}^c\text{G}^c) = P(\text{J}) - P(\text{JBG}^c) - P(\text{JB}^c\text{G}) - P(\text{JBG}) = 0.55 - 0.21 - 0.05 - 0.06 = 0.23$

   $P(\text{Bombers only}) = P(\text{J}^c\text{BG}^c) = P(\text{B}) - P(\text{JBG}^c) - P(\text{J}^c\text{BG}) - P(\text{JBG}) = 0.40 - 0.21 - 0.04 - 0.06 = 0.09$

   $P(\text{Goldeyes only}) = P(\text{J}^c\text{B}^c\text{G}) = P(\text{G}) - P(\text{JB}^c\text{G}) - P(\text{J}^c\text{BG}) - P(\text{JBG}) = 0.20 - 0.05 - 0.04 - 0.06 = 0.05$

   We know that the probability of all outcomes in the sample space must add up to one, so we find the probability that a randomly selected person isn't a fan of any of the three teams by adding the above probabilities (all probabilities inside the circles in the Venn diagram) and subtract the sum from one:

   $P(\text{J}^c\text{B}^c\text{G}^c) = 1 - (0.06 + 0.21 + 0.05 + 0.04 + 0.23 + 0.09 + 0.05) = 1 - 0.73 = 0.27$

5. $P(J)P(B) = (0.55)(0.40) = 0.22 \neq P(J \cap B)$

$\Rightarrow$ J and B are **not** independent.

$P(J)P(G) = (0.55)(0.20) = 0.11 = P(J \cap B)$

$\Rightarrow$ J and G **are** independent.

$P(B)P(G) = (0.40)(0.20) = 0.08 \neq P(B \cap G)$

$\Rightarrow$ B and G are **not** independent.

6. (a) The complete sample space of outcomes and the probabilities of each outcome are shown below:

Outcome	Probability
HH	$(0.7)(0.7) = 0.49$
HT	$(0.7)(0.3) = 0.21$
TH	$(0.3)(0.7) = 0.21$
TT	$(0.3)(0.3) = 0.09$

$P(X = 0) = P(TT) = 0.09$

$P(X = 1) = P(HT) + P(TH) = 0.21 + 0.21 = 0.42$

$P(X = 2) = P(HH) = 0.49$

The probability distribution of X is shown below:

x	0	1	2
$P(X = x)$	0.09	0.42	0.49

(b) There is a fixed number of trials ($n = 2$), there are only two outcomes of interest for each trial (Heads or Tails), trials are independent since the outcome of one coin toss doesn't affect the outcome of the next coin toss, and there is a constant probability of success ($p = 0.7$). It follows that X has a binomial distribution with parameters $n = 2$ and $p = 0.7$. Therefore,

$$P(X = 0) = \binom{2}{0}(0.7)^0(0.3)^2 = 0.09$$
$$P(X = 1) = \binom{2}{1}(0.7)^1(0.3)^1 = 0.42$$
$$P(X = 2) = \binom{2}{2}(0.7)^2(0.3)^0 = 0.49$$

7. If X follows a normal distribution, the sample mean $\overline{X}$ follows an exact normal distribution for any sample size n. So if $n = 5$ or $n = 50$, we can calculate an exact probability.

   If X does not follow a normal distribution and the sample size is low (< 30), we don't know the distribution of $\overline{X}$, so we cannot calculate a probability.

   If X does not follow a normal distribution, the Central Limit Theorem tells us that $\overline{X}$ will have an approximate normal distribution for large sample sizes (we use $n \geq 30$).

   Therefore, the correct answers are:

   - normal, 5, and exact
   - normal, 50, and exact
   - right-skewed, 50, and approximate

8. Note: The question you received may have had a different number of bricks than this solution. If so, your question will be marked accordingly.

   $$P(4.97 < \overline{X} < 5.04) = P\left(\frac{4.97 - 5}{0.3/\sqrt{45}} < Z < \frac{5.04 - 5}{0.3/\sqrt{45}}\right) = P(-0.67 < Z < 0.89)$$

   $$= P(Z < 0.89) - P(Z < -0.67) = 0.8133 - 0.2514 = 0.5619$$

9. $P(\text{Total} = 495) = P(\overline{X} = 4.95) = 0$ by definition, since $\overline{X}$ is a continuous variable.

10. Note: The question you received may have had a different number of bags of organic fertilizer than this solution. If so, your question will be marked accordingly.

    $$P(\overline{X} < 9.9) = P\left(Z < \frac{9.9 - 10}{0.5/\sqrt{38}}\right) = P(Z < -1.23) = 0.1093$$

11. Note: The question you received may have had a different number of bags of organic fertilizer than this solution. If so, your question will be marked accordingly.

    $$P(\text{Total} > 401.8) = P\left(\overline{X} > \frac{401.8}{41}\right) = P(\overline{X} > 9.8) = P\left(Z > \frac{9.8 - 10}{0.5/\sqrt{41}}\right)$$

    $$= P(Z > -2.56) = 1 - P(Z \leq -2.56) = 1 - 0.0052 = 0.9848$$

12. We want to find the value $\overline{x}$ such that $P(\overline{X} < \overline{x}) = 0.13$. First we find the value z such that $P(Z < z) = 0.13$:

$$P(Z < z) = 0.13 \quad \Rightarrow \quad z = -1.13$$

We now use the z-score formula to solve for $\overline{x}$ as follows:

$$z = \frac{\overline{x} - \mu}{\sigma/\sqrt{n}} \quad \Rightarrow \quad \bar{x} = \mu + z\left(\frac{\sigma}{\sqrt{n}}\right) = 10 + (-1.13)\left(\frac{0.5}{\sqrt{100}}\right) = 9.94$$

13. The 68-95-99.7 rule tells us that approximately 99.7% of values of a variable fall within three standard deviations of its mean. The mean and standard deviation of $\overline{X}$ are μ and $\sigma/\sqrt{n}$, respectively, so approximately 99.7% of values of $\overline{X}$ will fall within

$$\mu \pm 3\frac{\sigma}{\sqrt{n}} = 50 \pm 3\left(\frac{0.8}{\sqrt{4}}\right) = 50 \pm 1.2$$

i.e., between 48.8 and 51.2.

14. $P(\hat{p} < 0.05) \approx P\left(Z < \dfrac{0.05 - 0.08}{\sqrt{\dfrac{0.08(0.92)}{200}}}\right) = P(Z < -1.56) = 0.0594$

15. $P(X \geq 18) = P\left(\hat{p} \geq \dfrac{18}{200}\right) = P(\hat{p} \geq 0.09) \approx P\left(Z \geq \dfrac{0.09 - 0.08}{\sqrt{\dfrac{0.08(0.92)}{200}}}\right)$

$= P(Z \geq 0.52) = 1 - P(Z < 0.52) = 1 - 0.6985 = 0.3015$