Assignment 4 Solutions

1. A 97% confidence level is not given in Table 3, so we must use Table 2 to find the critical value z^*. We want to find the value z^* such that $P(-z^* < Z < z^*) = 0.97$. To use Table 2, we must find the total area to the left of z^*:

   $$P(Z < z^*) = P(Z < -z^*) + P(-z^* < Z < z^*) = \frac{1 - 0.97}{2} + 0.97 = 0.9850 \Rightarrow z^* = 2.17$$

2. A 97% confidence interval for the true mean IQ of all adult Canadians is

   $$\overline{x} \pm z^* \frac{\sigma}{\sqrt{n}} = 112 \pm 2.17 \left(\frac{15}{\sqrt{30}} \right) = 112 \pm 5.94 = (106.06, 117.94)$$

3. If we took repeated samples of 30 adult Canadians and calculated an interval in a similar manner, 97% of such intervals would contain the true mean IQ of all adult Canadians.

4-5. We are testing the hypotheses $H_0 : \mu = 105$ vs. $H_a : \mu \neq 105$.

   Since this is a two-sided test, and since the confidence level of the interval (97%) and the level of significance of the test (3%) add up to one, the confidence interval could be used to conduct the test. Since $\mu_0 = 105$ is not contained in the 97% confidence interval for μ, we reject H_0 at the 3% level of significance.

6. The null and alternative hypotheses must be stated in terms of parameters (like μ), not statistics (like $\overline{x}$). We're trying to find evidence that μ is greater than 110, so this statement goes into the alternative hypothesis. Then the null hypothesis is the same statement with an equal sign. The answer is:

   $$H_0 : \mu = 110 \text{ vs. } H_a : \mu > 110$$

7. We are told that IQs follow a normal distribution with standard deviation 15. This standard deviation is the population standard deviation σ. As such, we will conduct a z test. The test statistic is

   $$z = \frac{\overline{x} - \mu_0}{\sigma/\sqrt{n}} = \frac{112 - 110}{15/\sqrt{30}} = 0.73$$

8. The P-value is $P(Z > 0.73) = 1 - P(Z \leq 0.73) = 1 - 0.7673 = 0.2327$.

9. Since the P-value $> \alpha = 0.10$, we fail to reject H_0.

10. At the 10% level of significance, we have insufficient evidence that the true mean IQ of adult Canadians is greater than 110.

11. If the true mean IQ of adult Canadians were 110, the probability of observing a sample mean at least as high as 112 would be 0.2327.

12. Since we don't know the value of the population standard deviation σ, we will conduct a t test.

    We are testing the hypotheses $H_0 : \mu = 110$ vs. $H_a : \mu \neq 110$.

    The test statistic is

$$t = \frac{\overline{x} - \mu_0}{s/\sqrt{n}} = \frac{108 - 110}{5/\sqrt{30}} = -2.19$$

    The P-value is $2P(T(29) \leq -2.19) = 2P(T(29) \geq 2.19)$ by the symmetry of the t distributions. From Table 3, we see

$$P(T(29) \geq 2.150) = 0.02 \qquad \text{and} \qquad P(T(29) \geq 2.462) = 0.01$$

    Since $2.150 < 2.19 < 2.462$, it follows that $P(T(29) \geq 2.19)$ is between 0.01 and 0.02. But our P-value is double this probability, so the P-value is between 2(0.01) and 2(0.02), i.e., between 0.02 and 0.04.

13. Since we don't know the value of the population standard deviation σ, we will use the t distribution for the critical value in our confidence interval.

    The critical value t^* is the upper 0.025 critical value of the t distribution with $n - 1 = 29$ degrees of freedom. That is, $t^* = 2.045$.

14. We have reduced the margin of error by a factor of $\frac{5}{4} = 1.25$, so we require $(1.25)^2 = 1.5625$ times the original sample size. That is, we require a sample of size $100(1.5625) = 156.25 \approx 157$.