MATH 1300 D01 Assignment #1

Due: Thursday. September 29th, 2016

Instructions:

SHOW YOUR WORK to get full marks.

All assignments must be handed in on UMLearn as one PDF file. Late assignments will not be accepted. Failure to follow the instructions will result in a mark of 0.

This assignment covers all the topics in Unit 1, and well as Unit 2 topics found in sections 2.2-2.4.

The total number of marks for this assignment is 60.

1. (10 points) For each of the following matrices, identify if the matrix is in RREF. If it is not in RREF, give a condition of RREF that it fails to satisfy.

   (a) $\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

   (d) $\begin{pmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 1 & 0 & 0 \end{pmatrix}$

   (b) $\begin{pmatrix} 1 & 2 & 0 & -1 & 0 & 0 \\ 0 & 0 & 1 & 3 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 0 \end{pmatrix}$

   (e) $\begin{pmatrix} 1 & 7 & 0 & 4 & 0 & 0 \\ 0 & 0 & 3 & -2 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

   (c) $\begin{pmatrix} 1 & 0 & 1 & 0 \\ 0 & 1 & 0 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{pmatrix}$

   (f) $\begin{pmatrix} 1 & 0 & 12 & -5 & 0 & 7 \\ 0 & 1 & 3 & -2 & 0 & 6 \\ 0 & 0 & 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 & 0 & 0 \end{pmatrix}$

2. (10 points) Evaluate the following system using Gaussian elimination. When performing row reduction, be sure to indicate your row operations.

   (a) $\begin{aligned} x &+ 2y &- &z &= -3 \\ 2x &+ 5y &+ &z &= 1 \\ -x &+ 4y &- &2z &= 3 \end{aligned}$

   (b) $\begin{aligned} x_1 &+ 3x_2 &+ 3x_3 &- &x_4 &= 0 \\ 2x_1 &+ 6x_2 &+ 8x_3 &+ &3x_4 &= 0 \end{aligned}$

3. (10 points) Evaluate the following system using Gauss-Jordan elimination.When performing row reduction, be sure to indicate your row operations.

$$\begin{array}{rcl}
2x & - & y & + & z & = & 5 \\
x & + & y & + & 2z & = & 4 \\
-x & + & 3y & + & 2z & = & 0
\end{array}$$
(a)

$$\begin{array}{rcl}
x & - & y & + & z & = & 2 \\
2x & + & y & + & 3z & = & 1 \\
-x & - & 2y & - & z & = & 3
\end{array}$$
(b)

4. (12 points) Consider the following system of linear equations:

$$\begin{array}{rcl}
x & - & y & + & z & = & 1 \\
2x & + & y & - & z & = & 2 \\
3x & + & ay & + & bz & = & c
\end{array}$$

Where a, b and c are real values.

(a) Find the augmented matrix for this system.

(b) Reduce this matrix to as close to REF form as you can make it without knowing values for a, b, and c. (Hint: Never divide by zero.)

(c) Find all (real) values of a, b and c such that this system has no solutions.

(d) Find all (real) values of a, b and c such that this system has one solution.

(e) Find all (real) values of a, b and c such that this system has more than one solution.

5. (12 points) Using following matrices:

$A = \begin{pmatrix} 1 & 2 & 3 \\ 2 & -1 & 4 \end{pmatrix}$, $B = \begin{pmatrix} 3 & -2 & 4 \\ 7 & 3 & -5 \end{pmatrix}$, and $C = \begin{pmatrix} 1 & 1 \\ -1 & -1 \end{pmatrix}$,

evaluate the following if they are defined. If they are not defined, explain why not.

(a) $2A + B$

(b) $AB - C$

(c) $CB + A$

(d) $CA + 3B$

(e) $A + 2C$

6. (6 points) Given the matrices

$A = \begin{pmatrix} 1 & 2 \\ -1 & 1 \end{pmatrix}$, $B = \begin{pmatrix} 1 & 3 \\ -1 & -1 \end{pmatrix}$, and $C = \begin{pmatrix} 2 & 1 \\ 3 & -1 \end{pmatrix}$

is it possible to find values for the scalars x, y and z so that : $xA + yB + zC = \begin{pmatrix} 6 & 9 \\ 1 & -3 \end{pmatrix}$.