MATH 1300 D01 Assignment #2

Due: Monday, October 17^{th}, 2016

Instructions:

SHOW YOUR WORK to get full marks.

All assignments must be handed in on UMLearn as **one** PDF file. Late assignments will not be accepted. Failure to follow the instructions will result in a mark of 0.

This assignment covers topics to the middle of Unit 3 (specifically section 3.4), the focus is on topics found in sections 2.5 - 2.12 and 3.2 - 3.4.

The total number of marks for this assignment is 70.

We define the matrices to be used in question 1 and 2 (only):

$A = \begin{pmatrix} 1 & -3 & 1 \\ 3 & 1 & -2 \end{pmatrix}$, $B = \begin{pmatrix} 1 & -1 \\ 3 & 2 \\ 4 & -1 \end{pmatrix}$, and $C = \begin{pmatrix} 1 & 2 \\ -2 & -1 \end{pmatrix}$, $D = \begin{pmatrix} 1 & 2 & -3 \\ 2 & -1 & 4 \\ 1 & 3 & 1 \end{pmatrix}$, and $F = \begin{pmatrix} 1 & 2 & -3 \\ 2 & -1 & 4 \end{pmatrix}$

1. (18 points) Evaluate the following if they are defined. If they are not defined, explain why not.
   (a) $AF^T + C$
   (b) $AD - (FC)^T$
   (c) $C^T A + B^T D$
   (d) $C(F + B^T)D$
   (e) $B(AF + I_2)$
   (f) $DA + DB^T + DF$

2. (12 points) Evaluate the following if they are defined. If they are not defined, explain why not.
   (a) trace(D)
   (b) trace(BC)
   (c) trace(AB)
   (d) trace(BA)

3. (8 points) Given the following system of linear equations

$$\begin{array}{rcrcrcr} x & + & 3y & - & z & = & 9 \\ -x & - & 2y & + & 4z & = & -19 \\ 2x & + & 7y & + & 2z & = & 4 \end{array}$$

(a) Write the system of linear equations in matrix form.

(b) Find the inverse of the coefficient matrix.

(c) Use the inverse to solve the system of linear equations.

4. (8 points) Given the following system of linear equations

$$\begin{array}{rcrcrcr} x & + & 4y & + & z & = & 0 \\ 3x & + & 7y & + & 2z & = & 0 \\ 7x & - & 2y & + & z & = & 0 \end{array}$$

(a) Write the system of linear equations in matrix form.

(b) Show that the coefficient matrix is not invertible.

(c) How many solutions does this system of equations have (you need not find any solutions to answer this question).

5. (12 points) Given $A = \begin{pmatrix} 1 & 0 & 1 \\ 0 & 1 & 2 \\ 1 & -1 & 2 \end{pmatrix}$:

(a) Write A as the product of elementary matrices. (Note that this product is not unique.)

(b) Find the inverse of A.

(c) Write A^{-1} as the product of elementary matrices.

6. (12 points) Find the determinant of the given matrix. (Use the given method if one is specified.)

(a) $A_1 = \begin{pmatrix} 2 & -1 & 2 \\ 1 & 1 & 4 \\ -1 & 3 & 4 \end{pmatrix}$.

(b) $A_2 = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 5 & 4 \\ -1 & 1 & -2 \end{pmatrix}$, cofactor expansion along 1^{st} row.

(c) $A_2 = \begin{pmatrix} 1 & 2 & -1 \\ 2 & 5 & 4 \\ -1 & 1 & -2 \end{pmatrix}$, cofactor expansion along 3^{rd} column.

(d) $A = \begin{pmatrix} -1 & 3 & 1 & -2 \\ 0 & 0 & 0 & 7 \\ 0 & 2 & -1 & 2 \\ 0 & 0 & 3 & -2 \end{pmatrix}$.

(e) $A = \begin{pmatrix} 1 & 4 & -1 & 2 \\ -1 & 3 & 0 & -2 \\ 1 & 0 & 0 & 7 \\ -1 & 0 & 2 & -2 \end{pmatrix}$, (use elementary row operations).