MATH 1300 D01 Assignment #3

Due: Monday, November 14^{th}, 2016

Instructions:

SHOW YOUR WORK to get full marks.

All assignments must be handed in on UMLearn as **one** PDF file. Late assignments will not be accepted. Failure to follow the instructions will result in a mark of 0.

This assignment covers topics from Unit 4 and Unit 5.

The total number of marks for this assignment is 75.

For this assignment, I will use angled brackets for vectors to distinguish them from points. For example $\mathbf{u} = \langle 3, 1, 2\rangle$ is a vector but $P = (3, 1, 2)$ is a point. This convention is not used in the notes, and may not be used for the final exam.

1. (32 points) Let $\mathbf{u} = \langle 1, -1, 2\rangle$, $\mathbf{v} = \langle 3, 1, -2\rangle$, and $\mathbf{w} = \overrightarrow{PQ}$ where $P = P(1, 2, 3)$ and $Q = Q(3, 3, 1)$.

   Compute the following:

   (a) components of $\mathbf{w}$.

   (b) $3\mathbf{u} - 2\mathbf{v}$

   (c) $|| - 2\mathbf{u}||$

   (d) a unit vector in the direction of $\mathbf{w}$.

   (e) $\mathbf{u} \cdot \mathbf{v}$

   (f) $\mathbf{v} \times \mathbf{u}$

   (g) $(\mathbf{u} \cdot \mathbf{v}) \times \mathbf{w}$

   (h) $(\mathbf{u} + \mathbf{w}) \times \mathbf{v}$

   (i) the area of the triangle having two of its sides the vectors $\mathbf{u}$ and $\mathbf{v}$.

   (j) the volume of the parallelepiped with sides the vectors $\mathbf{u}$,$\mathbf{v}$, and $\mathbf{w}$.

   (k) the projection of $\mathbf{v}$ onto $\mathbf{w}$ ($\text{proj}_{\mathbf{w}}(\mathbf{v})$).

   (l) the vector component of $\mathbf{u}$ orthogonal to $\mathbf{v}$.

2. (10 points) Let $\mathbf{v} = \langle 3, -1, 2\rangle$ and $\mathbf{v} = \langle 1, 4, 1\rangle$ and let θ be the angle between them.

   (a) Using the formula with the cross product, find $\sin(\theta)$.

   (b) Using the formula with the dot product, find $\cos(\theta)$.

   (c) Verify your answers with the identity $\sin^2(\theta) + \cos^2\theta = 1$.

3. (15 points)

   (a) Find a standard equation of a plane that passes through the points $A(1, 1, 1)$, $B(2, 3, 4)$ and $C(-1, 1, -2)$.

   (b) Find a standard equation of a plane that contains the lines $x = 1 + t$, $y = 2 - 3t$, $z = -1 + 2t$, and $x = 5 - s$, $y = -7 + 2s$, $z = 4 - s$.

   (c) Find a standard equation of a plane that contains the lines $x = 1 + t$, $y = 2 - 3t$, $z = -1 + 2t$, and $x = 4 + 2s$, $y = 2 - 6s$, $z = -5 + 4s$.

   (d) Find the distance from the point $P(1, 2, 3)$ and the plane $7x + 4y - 2z = 10$.

4. (6 points) Find parametric equations of a line that passes through point $P(-2, 1, 3)$ and is perpendicular of line $x = 2 + 3t$, $y = 4 - t$, $z = -1 + 4t$.

5. (6 points) Show that for vectors $\mathbf{u}$, $\mathbf{v}$,and $\mathbf{w}$ in $\mathbb{R}^3$,

$$||\mathbf{u} - \mathbf{w}|| \leq ||\mathbf{u} - \mathbf{v}|| + ||\mathbf{v} - \mathbf{w}||.$$

6. (6 points) Show that for vectors $\mathbf{u}$ and $\mathbf{v}$ in $\mathbb{R}^3$, if $||\mathbf{u}|| = ||\mathbf{v}||$ then the vectors $(\mathbf{u} + \mathbf{v})$ and $(\mathbf{u} - \mathbf{v})$ are orthogonal.