# MATH 1300 ASSIGNMENT PROBLEMS (UNIT 1)

Please refer to UMLearn for instructions on submitting your assignment. The "How to Submit" link will provide detailed instructions. Your assignment must be uploaded in 1 pdf file. Failure to do so will result in your assignment receiving a mark of zero.

1. AOB is the diameter of a circle with centre at O and C is any other point on the circle. Denote the vector $\overrightarrow{OA}$ by **a** and the vector $\overrightarrow{OC}$ by **c.**
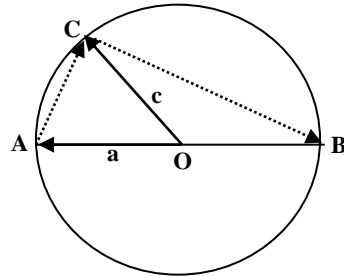
[4]   (a) Write the vectors $\overrightarrow{AC}$ and $\overrightarrow{CB}$ in terms of the vectors **a** and **c.**

[6]   (b) Use vector methods to show that $\angle ACB$ is a right angle.

2. Let A = (−1, 2, 3) and B = (1, 4, −3) be two points in $\mathbf{R}^3$.

[2]   (a) Find the components of the vectors $\overrightarrow{AB}$ and $\overrightarrow{BA}$.

[2]   (b) Find the coordinates of the midpoint of the line segment AB.

[3]   (c) The point C = (k, 9, 4) is equidistant from the points A and B. Find the value(s) of k.

[3]   (d) If $\overrightarrow{AX}$ = (−1, 2, 3), find the coordinates of the point X.

3. Let $\mathbf{u} = (1, -2, 3)$, $\mathbf{v} = (2, -3, 4)$ and $\mathbf{w} = (4, -1, 5)$ be three vectors in $\mathbf{R}^3$. Find the following.

[2]   (a) $2\mathbf{u} + 3\mathbf{v} - 4\mathbf{w}$

[2]   (b) $\mathbf{u} \cdot \mathbf{v}$

[2]   (c) $\mathbf{u} \times \mathbf{w}$

[2]   (d) $\text{proj}_\mathbf{v}\mathbf{u}$

[2]   (e) cosine of the angle between the vectors **u** and **w**.

4. Let $\mathbf{u} = (4, 2, 7)$ and $\mathbf{v} = (2, 1, k)$ be two vectors in $\mathbf{R}^3$.

[3]   (a) For what value(s) of k will the two vectors $\mathbf{u}$ and $\mathbf{v}$ be parallel? Explain.

[3]   (b) For what value(s) of k will the two vectors $\mathbf{u}$ and $\mathbf{v}$ be orthogonal? Explain.

[4]   (c) For what value(s) of k will the two vectors $\mathbf{u}$ and $\mathbf{v}$ be of equal length?

5. Let l: $2x + 3y = 6$ be a line and P $= (5, 3)$ be a point in $\mathbf{R}^2$.

[3]   (a) Let Q be the point on the line l having its y-coordinate $= 0$ and let R be the point on the line l having its x-coordinate $= 0$. Find the coordinates of the points Q and R.

[3]   (b) Plot the points P, Q, R and the line l on a two-dimensional Cartesian coordinate system.

[2]   (c) Find the components of the vector $\overrightarrow{QP}$.

[2]   (d) Find a normal vector $\mathbf{n}$ to the given line l.

6. The plane $x + 2y + 2z = 4$ intersects the positive coordinate axis OX, OY and OZ in three points A, B and C respectively.

[3]   (a) Find the coordinates of the three points A, B and C.

[7]   (b) Find the area of the triangle ABC.

plane $x + 2y + 2z = 4$