MATH 1300 ASSIGNMENT PROBLEMS (UNIT 3)

[10] 1. Determine into which of the following 3 types (A), (B) or (C) the matrices (a) to (e) below can be classified.

Type (A): The matrix is in both reduced row-echelon form and row-echelon form.
Type (B): The matrix is in row-echelon form but not in reduced row-echelon form.
Type (C): The matrix is in neither reduced row-echelon form nor in row-echelon form.

$$\text{(a)}\begin{bmatrix}1&0&1&0\\0&0&1&0\\0&0&0&1\\0&0&0&0\end{bmatrix}\quad \text{(b)}\begin{bmatrix}1&1&0&0\\0&0&1&1\\0&0&0&0\\0&0&0&0\end{bmatrix}\quad \text{(c)}\begin{bmatrix}1&0&0&0\\0&0&1&0\\0&1&0&0\\0&0&0&1\end{bmatrix}\quad \text{(d)}\begin{bmatrix}1&2&0&3\\0&0&1&4\\0&0&0&1\\0&0&0&0\end{bmatrix}\quad \text{(e)}\begin{bmatrix}0&0&0&0\\0&0&0&0\\0&0&0&0\\0&0&0&0\end{bmatrix}$$

(f) Find all 3×2 reduced row-echelon matrices. (Hint: there are 4 possible answers)

[10] 2. Consider the following system of linear equations.

$$\begin{aligned} x_1 + x_3 + x_4 &= 5\\ x_1 + x_2 - x_3 &= 2\\ x_2 + x_3 + x_4 &= 3\\ x_1 - x_2 + x_4 &= 2 \end{aligned}$$

(a) Write out the augmented matrix for this system of linear equations.

(b) Use elementary row operations to reduce the augmented matrix to reduced row-echelon form.

(c) Write out the solution to the system of linear equations.

[10] 3. The augmented matrix from a system of linear equations has the following reduced row-echelon form.

$$\left[\begin{array}{ccccccc|c}1&3&0&5&0&4&0&2\\0&0&1&2&0&2&0&3\\0&0&0&0&1&3&0&4\\0&0&0&0&0&0&1&5\\0&0&0&0&0&0&0&0\end{array}\right]$$

(a) How many equations are there in the system?

(b) How many variables are there in the system?

(c) How many of the variables are independent variables?

(d) Write out the solution set for the system.

[10] 4. Consider the system of linear equations

$$\begin{aligned} x + 3y &= 5 \\ 2x + ay &= b \end{aligned}$$

where a and b are real numbers.

(a) Write out the augmented matrix for this system of linear equations.

(b) Use elementary row operations to reduce the augmented matrix to row-echelon form.

(c) Determine for what values of a and b does the system have infinitely many solutions.

(d) Determine for what values of a and b does the system have no solution.

(e) Determine for what values of a and b does the system have an unique solution.

[10] 5. Michael, John and Matthew went to their local grocery store. Michael bought two kilograms of almonds and one kilograms of peanuts and paid $20.00. John bought one kilogram of almonds and two kilograms of cashews and paid $24.00. Matthew bought three kilograms of peanuts and one kilogram of cashews and paid $25.20.

(a) Let x = price of a kilogram of almonds, y = price of a kilogram of peanuts and z = price of a kilogram of cashews. Write out 3 linear equations representing the purchases made by Michael, John and Matthew.

(b) Write out the augmented matrix for your system of 3 linear equations of part (a).

(c) Use elementary row operations to row reduce the augmented matrix of part (b) to a reduced row-echelon matrix.

(d) What is the price per kilogram for each of these items?

[10] 6. Consider the linear equation $ax+by+cz=d \quad (d \neq 0) \quad (1)$

and the associated homogeneous equation $ax+by+cz=0 \quad (2)$.

Let (x_0, y_0, z_0) and (x_1, y_1, z_1) be two solutions to equation (1) and let (x_2, y_2, z_2) be a solution to equation (2).

(a) Show that $(x_0 - x_1,\ y_0 - y_1,\ z_0 - z_1)$ is a solution to equation (2).

(b) Show that $(x_0 + x_2, y_0 + y_2, z_0 + z_2)$ is a solution to equation (1).

(c) Let k be any real number. Show that (kx_2, ky_2, kz_2) is a solution to equation (2).