

MATH 1300 D01/D02 Winter 2016 Assignment 4

SHOW ALL WORK to get full marks. Leave answers as exact answers. For example, leave it as fractions such as $1/7$ as opposed to decimals such as 0.142857. Word problems should have sentence answers with units. Fractions should be lowest terms.

1. Let $A = \begin{bmatrix} 6 & 1 \\ 3 & -4 \end{bmatrix}$, $B = \begin{bmatrix} -7 & 1 & 2 \\ 0 & -2 & 4 \end{bmatrix}$, $C = \begin{bmatrix} 2 & 1 & -3 \\ 3 & 0 & 5 \\ 2 & -5 & 1 \end{bmatrix}$ evaluate the following, if possible. If not, explain why.

[4] (a) $AB - 5B$.

[2] (b) $(A - 5)B$.

[2] (c) CB .

[5] (d) $CB^T - (AB)^T$

[3] 2. (a) Find non-zero 2×2 matrices A and B such that $AB = O$.

[3] (b) Show that if B is invertible, and $AB = O$, then $A = O$.

Note: (b) does not depend on (a) and the O is the zero matrix, not the number 0.

3. For the following system of equations

$$2x + 3y + 4z = -3$$

$$x + 2y + 3z = 2$$

$$y + 3z = 4$$

[1] (a) Write the coefficient matrix A .

[6] (b) Use row operations to find A^{-1} .

[3] (c) Use the inverse to solve the system.

4. The matrix $A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -4 & 0 & 12 \end{bmatrix}$ can be reduced down to the identity in the following three steps

$$A = \begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ -4 & 0 & 12 \end{bmatrix} \Rightarrow \begin{bmatrix} -4 & 0 & 12 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & -3 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

[3] (a) What are the three elementary row operations used in order.

[3] (b) Using part (a), find elementary matrices E_1, E_2 and E_3 such that $E_3E_2E_1A = I$.

[2] (c) Write A^{-1} as a product of elementary matrices. Leave your answer as the product. Do not multiply.

[3] (d) Write A as a product of elementary matrices. Leave your answer as the product. Do not multiply.

5. Let $A = \begin{bmatrix} 5 & -3 \\ 6 & -4 \end{bmatrix}$, $D = \begin{bmatrix} -1 & 0 \\ 0 & 2 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 1 \\ 2 & 1 \end{bmatrix}$.

[4] (a) Compute P^{-1} and verify that $A = PDP^{-1}$.

[2] (b) What is the mistake in the following argument?

" $A = D$ since $A = PDP^{-1} = PP^{-1}D = ID = D$."

[4] (c) Use part (a) to compute A^{2016} .

6. Suppose there are 3 internet companies, Net A, Net B and Net C. After every month, anybody can choose to stay with their company, or switch to one of the other two. (For simplicity sake, lets assume that no new customers come in, nor any current customer cancel their internet completely.)

Data shows after each month 80% of Net A customers stay, 10% switch to Net B and 10% switch to Net C. For Net B customers, 70% of customers stay, 20% switch to Net A and 10% switch to Net C. For Net C customers, 80% of people stay, 15% switch to Net A and 5% switch to Net B.

[2] (a) Write down the transition matrix. (First column for A, second column for B and third column for C)

[8] (b) After a very long time, what percentage will have Net A, what percentage will have Net B and what percentage will have Net C.

This assignment is out of 60 points.