

MATH 1300 ASSIGNMENT PROBLEMS (UNIT 5)

- [10] 1. Use a third row cofactor expansion to evaluate the determinant of

$$A = \begin{bmatrix} 5 & 1 & 4 & 1 \\ 4 & 1 & 2 & 5 \\ 2 & 3 & 4 & 1 \\ 1 & 1 & 2 & 4 \end{bmatrix}.$$

- [10] 2. Compute the determinant of $A = \begin{bmatrix} 4 & 1 & 5 & 2 \\ 2 & 4 & 6 & 8 \\ 4 & 5 & 1 & 3 \\ 5 & 6 & 3 & 1 \end{bmatrix}$ by the method of using elementary row operations to transform the matrix A into an upper triangular matrix.

[10] 3. Let $A = \begin{bmatrix} 1 & 3 & 0 \\ 4 & 2 & 1 \\ 1 & 3 & 2 \end{bmatrix}$.

- Find the matrix $\text{adj}(A)$.
- Compute the matrix product $A \cdot \text{adj}(A)$.
- Use the information from part (b) to determine the value of $\det(A)$. Explain.
- Use the information from parts (a) and (c) to find the inverse matrix A^{-1} .

- [10] 4. Use Cramer's rule to solve the following system of linear equations.

$$\begin{aligned}x + y + z &= 4 \\2x - y + 2z &= 4 \\3x + 2y + z &= 6\end{aligned}$$

[10] 5. Let A and B be 3×3 matrices with $\det(A) = 4$ and $\det(B) = 7$. Determine the values of the following.

(a) $\det(AB)$

(b) $\det(A^T B^{-1})$

(c) $\det((A^T B)^{-1})$

(d) $\det(2A)$

(e) $\det(\text{adj}(A))$

[10] 6(a) Let $A = \begin{bmatrix} k+1 & 4 \\ 1 & k-2 \end{bmatrix}$. For what values of k is the matrix A noninvertible?

(b) Let $B = \begin{bmatrix} 1 & 1 & 1 \\ 2 & k & 6 \\ 3 & 2 & 1 \end{bmatrix}$. For what values of k is the matrix B noninvertible?