MATH 1300 – D01 – Assignment 1

Solutions

Remarks, in somewhat random order.

1. Multiple columns. Some of you will have had remarks about presenting your answers using multiple columns. Please avoid this, in particular in assignments: you have plenty of space (essentially, unlimited), so use more pages rather than muddle things up by splitting them between columns.
2. Orientation: this time, I have not taken any points away for scans appearing sideways or upside down. Next time, I might not be so happy about having to do yoga to read your production.
3. Matrices are delimited by brackets or parentheses. I automatically give a zero to a question where a matrix is present and those delimiters are absent.
4. You need to "say" things to make your answers understandable. For instance, in question 2, someone answering "a) neither, b) RREF, c) REF, d) RREF" has, technically, the correct answer but will not get full marks. This is actually to your benefit: an incorrect answer is worth nothing, whereas an incorrect but motivated answer will very likely get part marks.

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1. Write the augmented matrix corresponding to the following system [5]

$$\begin{array}{ccccl} 2x_1 & +3x_2 & -2x_3 & +x_4 & = 2 \\ 3x_1 & & -x_3 & +4x_4 & = 3 \\ -2x_1 & +2x_2 & -x_3 & -x_4 & = 0 \\ x_1 & +x_2 & +x_3 & +x_4 & = 2 \end{array}$$

Solution: The augmented matrix associated to the system is

$$\left[\begin{array}{cccc|c} 2 & 3 & -2 & 1 & 2 \\ 3 & 0 & -1 & 4 & 3 \\ -2 & 2 & -1 & -1 & 0 \\ 1 & 1 & 1 & 1 & 2 \end{array}\right]$$

Note: You did not need to compute the solution. Please provide a simple sentence (as above or even shorter) to put things in context.

2. State whether the following matrices are in row-echelon form (REF), reduced row-echelon form (RREF), both or neither. [10]

$$(a)\quad \begin{bmatrix} 0 & 0 \\ 1 & 0 \end{bmatrix}, \qquad (b)\quad \begin{bmatrix} 0 & 1 \\ 0 & 0 \end{bmatrix}, \qquad (c)\quad \begin{bmatrix} 1 & 1 & 4 \\ 0 & 0 & 1 \end{bmatrix} \qquad (d)\quad \begin{bmatrix} 1 & 1 & -1 \\ 0 & 0 & 0 \end{bmatrix}$$

Solution: a) Neither, since there is a row of zeros above a row with a leading one.

b) RREF, as the top row has a leading one and the second row has all zeros.

c) Both rows have leading ones, the second row has its leading one to the right of that of the row above, so REF. The leading one in the second row is not the only nonzero entry in this column, so not RREF.

d) First row has leading one, second row is all zeros and comes below any row with a leading one, so RREF.

Note: Of course, RREF implies REF, so you could just say RREF when it was both. Providing minimal explanations (as above) was needed for full marks.

3. Consider the system [10]

$$\begin{aligned} x + ay &= b \\ x - y &= 2, \end{aligned}$$

where $a, b \in \mathbb{R}$. Write the augmented matrix corresponding to the system. Solve the system by substitution and by elimination. Find values of a, b such that the system has a) no solution, b) a unique solution and c) infinitely many solutions; plot the situation in each of these cases.

Solution: The augmented matrix associated to the system is

$$\left[\begin{array}{cc|c} 1 & a & b \\ 1 & -1 & 2 \end{array}\right]$$

Solving by substitution: from the second equation, $x = y + 2$, which substituted into the first equation gives

$$(y + 2) + ay = b.$$

Solving for y,

$$y = \frac{b-2}{1+a},$$

provided $a \neq -1$. In turn,

$$x = \frac{b-2}{1+a} + 2 = \frac{2a+b}{1+a}.$$

So the solution is

$$(x, y) = \left(\frac{2a+b}{1+a}, \frac{b-2}{1+a}\right).$$

Solving by elimination: using the augmented matrix

$$\left[\begin{array}{cc|c} 1 & a & b \\ 1 & -1 & 2 \end{array}\right] \xrightarrow{R_2 \leftarrow R_2 - R_1} \left[\begin{array}{cc|c} 1 & a & b \\ 0 & -1-a & 2-b \end{array}\right] \xrightarrow{R_2 \leftarrow R_2/(-1-a)} \left[\begin{array}{cc|c} 1 & a & b \\ 0 & 1 & \dfrac{b-2}{1+a} \end{array}\right]$$

$$\xrightarrow{R_1 \leftarrow R_1 - aR_1} \left[\begin{array}{cc|c} 1 & 0 & b - a\dfrac{b-2}{1+a} \\ 0 & 1 & \dfrac{b-2}{1+a} \end{array}\right]$$

Note that this assumes $1 + a \neq 0$, i.e., $a \neq -1$. Since

$$b - a\frac{b-2}{1+a} = \frac{b(1+a) - a(b-2)}{1+a} = \frac{2a+b}{1+a},$$

the solution is

$$(x, y) = \left(\frac{2a+b}{1+a}, \frac{b-2}{1+a}\right).$$

Different cases. You just needed to find values, so just pick some that work. a) no solution: take $a = -1$ and $b = 0$, so that the two lines are parallel but separated. b) a unique solution: take $a = 0, b = 0$. c) infinitely many solutions: take $a = -1$ and $b = 2$ so that the lines are parallel and overlapping. See Figure 1.

Note: Answer questions in the order they come.

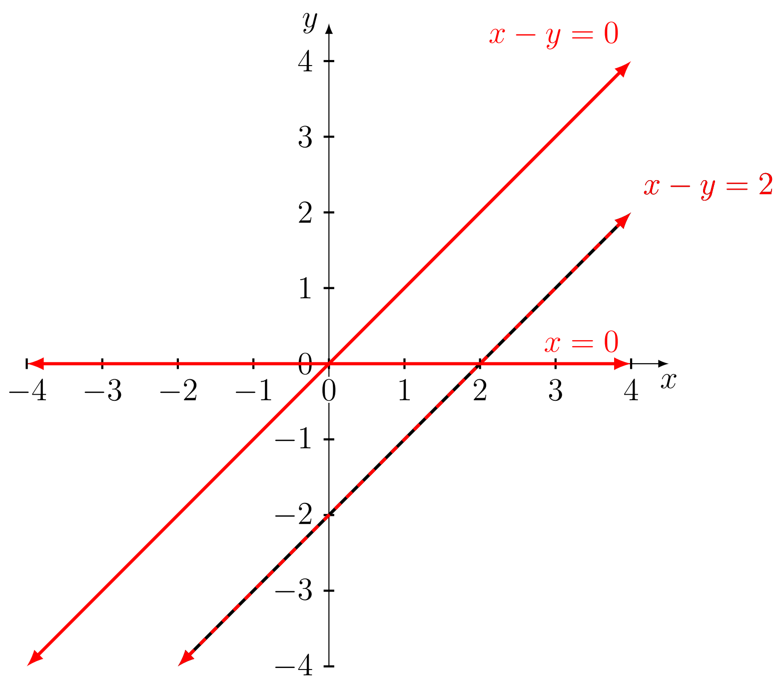

Figure 1: The curves chosen as examples in Exercise 3. (Ignore the arrow heads, I have not managed to remove them yet.)