

MATH 1300 – D01 – Assignment 3

Due Monday 2017-11-06 at 23:59

Student number: _____

Surname: _____

First name: _____

There are 5 exercises (check the back of this page for 2 more). Answer the following questions **on separate sheets** and scan the result to produce a single PDF file. Please show your work. Unclear or not fully justified answers **will not** get full marks.

1. Consider the points $A(3, -2, 1)$ and $B(0, 1, 2)$, the lines $l_1 = (t - 1, 2t + 2, -2t)$ and $l_2 = (-2s + 1, s - 1, 3s + 2)$ (for $s, t \in \mathbb{R}$), and the planes $\Pi_1 : 2x - y + 2z = 4$ and $\Pi_2 : -3x + 2y - z = 3$. Find the following:

1. Equation for the plane through B and perpendicular to l_1 .
2. Parametric equation of the line through A and B .
3. Intersection of the line l_2 with the plane Π_1 .
4. Point normal equation of the plane through B that is perpendicular to both Π_1 and Π_2 .
5. Intersection of l_1 and l_2 .
6. Equation for the plane containing A and l_1 .

2. Find all vectors $v = (x, y, z)$ in 3-space orthogonal to the vector $a = (0, 0, 1)$ and [10] such that $\|v\| = 1$. Draw a picture.

3. Given that

$$A = \begin{bmatrix} 5 & 2 \\ 0 & k \end{bmatrix},$$

find **all** values of k for which $A^2 - 7A + 10I_2 = 0_{22}$, where I_2 is the 2×2 identity matrix and 0_{22} is the 2×2 zero matrix.

4. It is given that A and B are two 4×4 matrices.
1. Write $\det(2A^{-1})$ in terms of $\det(A)$.
 2. Write $\det(A^2(B^{-1})^T)$ in terms of $\det(A)$ and $\det(B)$.
 3. Use part (1) and part (2) to calculate $\det(A)$ and $\det(B)$ if $\det(2A^{-1}) = 8 = \det(A^2(B^{-1})^T)$.

5. Let A be an $n \times n$ matrix such that $A^2 = A$. (Such matrices are called idempotent.) Let I_n denote the identity $n \times n$ matrix.

1. Show that $(I_n - 2A)^{-1} = I_n - 2A$.
2. Show that if A is such that $A^2 = A$, and the homogeneous system $AX = 0$ has only the trivial solution, then $A = I_n$.