Student number: _____

Surname: _____

First name: _____

This assignment consists of FOUR pages. Show your solutions.

1. Let A, B and C be the following matrices:

$$A = \begin{bmatrix} 1 & 2 & 3 \\ 3 & 2 & 1 \end{bmatrix}, \qquad B = \begin{bmatrix} 1 & 2 \\ 3 & 4 \end{bmatrix}, \qquad C = \begin{bmatrix} 0 & 1 & 2 \\ 4 & 0 & 4 \\ 2 & 1 & 0 \end{bmatrix}.$$

Compute the following matrices or state why the matrix is not defined.

(a) $A^T A + C$

(b) $CA^T + BA$

(c) $\frac{1}{4}C(BA)^T$

(d) $2B^{-1}A + AC$

2. Consider the following matrix,

$$M = \begin{bmatrix} 1 & 1 & 1 \\ 2 & 3 & 4 \\ 0 & 2 & 2 \end{bmatrix}.$$

(a) Compute M^{-1} by augmenting M with a 3×3 identity matrix and using row reduction.

(b) Write M and M^{-1} as the product of elementary matrices.

(c) Calculate $\det(M)$ by cofactor expansion.

(d) Calculate $\det(M)$ by using row reduction to a triangular matrix (see unit 3.4).

3. Consider the following system of equations.

$$x + 2y = a$$
$$a^2x + 2y = 1$$

(a) How many solutions does this system have if $a = -1$?

(b) How many solutions does this system have if $a = 1$?

(c) How many solutions does this system have if $a \neq 1$ and $a \neq -1$?