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This assignment consists of THREE pages. Show your solutions.

1. Find the standard matrix corresponding to the following linear transformations. Evaluate all trigonometric expressions.

(a)  $T(x, y, z) = (x - z, x + y - \frac{z}{2}, 4y - 3z)$

(b) Rotation on  $\mathbb{R}^2$  by  $\frac{\pi}{3}$  clockwise

(c) Reflection on  $\mathbb{R}^2$  about the  $y$ -axis

(d) Reflection on  $\mathbb{R}^2$  about the line  $y = x$

2. Consider a linear transformation  $T : \mathbb{R}^2 \rightarrow \mathbb{R}^2$  such that

$$T(\vec{e}_1 - \vec{e}_2) = (-1, -2), \quad T(3\vec{e}_2) = (0, 6).$$

Use the properties of linear transformations to find the following:

(a) Find  $T(\vec{e}_2)$ .

(b) Find  $T(\vec{e}_1)$ .

(c) Find the standard matrix associated with  $T$ .

(d) Evaluate  $T\left(\begin{bmatrix} -1 \\ 0 \end{bmatrix}\right)$ .

(e) Find the standard matrix associated with  $T^{-1}$ .

(f) Apply  $T^{-1}$  to your answer in part (d) and verify that you get  $\begin{bmatrix} -1 \\ 0 \end{bmatrix}$  back.

3. Consider the following matrix

$$A = \begin{bmatrix} 2 & -2 \\ -3 & 1 \end{bmatrix}.$$

(a) Find the eigenvalues of  $A$ .

(b) Find the eigenvectors corresponding to the eigenvalues in part (a).

(c) Give a diagonal matrix  $D$  and an invertible matrix  $P$  such that  $A = PDP^{-1}$ .

(d) Compute  $P^{-1}$  and verify that  $A = PDP^{-1}$  (show your steps).