MATH 1500 D01 Fall 2016 Assignment 2

SHOW ALL WORK to get full marks. Leave answers as exact answers. For example, leave it as 1/7 as opposed to 0.142857. Word problems should have sentence answers with units. This assignment covers sections from 2.7, 2.8, 3.1–3.5, 3.9.

Use the definition of derivative only for Q1. Do not use the definition in any other questions.

Word problems should have sentence answers with units.

[7] 1. Use the definition of the derivative to calculate the derivative of the function $f(x) = \dfrac{1}{\sqrt{x^2+5}}$.

2. Calculate the derivative of the following functions. Do not simplify your answers.

[3] (a) $f(x) = \sqrt[3]{x} + \dfrac{2}{x^{2016}} + \dfrac{1}{\sqrt{x^3}}$

[4] (b) $f(x) = \dfrac{4 - xe^x}{x + e^x}$

[5] (c) $f(x) = e^{e^{5\tan x}}$

[5] (d) $f(x) = \cos(\sqrt{e^{2x} + \cot x})$

3. Evaluate the following limits. Show all steps.

[4] (a) $\lim\limits_{x\to 0} \dfrac{2\sin 3x}{\sin 5x}$

[4] (b) $\lim\limits_{t\to 2} \dfrac{3\sin(t-2)}{(t^2-6t+8)}$

4. If a rock is thrown upwards with a velocity of 15 metres per second, its height, in metres, is given by

$$H(t) = -4.9t^2 + 15t.$$

(Calculus method and not physics methods must be used)

[1] (a) Find the velocity at any time t seconds.

[3] (b) After how many seconds will the rock stop rising and begin to fall?

[4] (c) After what time, and with what velocity will the rock hit the ground?

[5] 5. Calculate an equation of the tangent line to $f(x) = (x^3 - 2x - 1)e^x$ when $x = 0$.

[4] 6. If $f(2) = 4$ and $f'(2) = -3$, calculate $g'(2)$ if $g(x) = \dfrac{f(x)}{x}$.

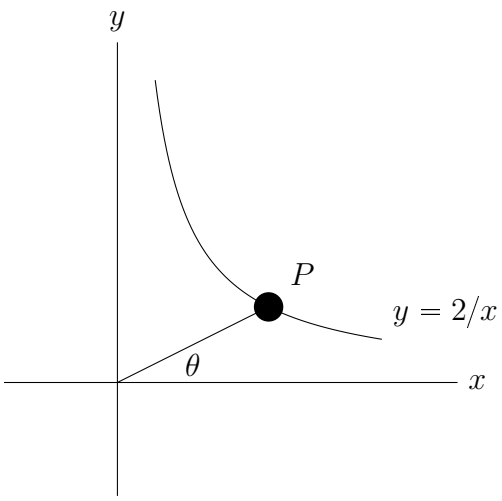
[8] 7. Use implicit differentiation to find the equation of the tangent line to the curve

$$x^3 - \tan xy + y^3 = e^y - 1$$

at the point $(1, 0)$.

[9] 8. A spotlight on the ground shines on a wall 12 metres away. A person 2 metres tall walks from the spotlight towards the building at a speed of 4/5 metres per second. How fast is the length of their shadow on the building changing when they are 4 meters from the building?

[9] 9. A particle P is moving along the curve $y = \dfrac{2}{x}$ so that its x coordinate (in meters) is increasing at a rate of 2 m/s. The line segment between the origin (0, 0) and P forms an angle θ between the line segment and the positive x-axis. See the figure below



Compute the rate of change of θ with respect to time when the particle passes through $x = 2$. (Hint: You may wish to use $\sec^2\theta = 1 + \tan^2\theta$ at some point.)

This assignment is out of 75 points.