MATH 1500 D01 Fall 2016 Assignment 4

SHOW ALL WORK to get full marks. Leave answers as exact answers. For example, leave it as fractions such as 1/7 as opposed to decimals such as 0.142857. Word problems should have sentence answers with units. Fractions should be lowest terms.

All assignments must be handed in on UMLearn as **one PDF file**. Late assignments will not be accepted. Failure to follow the instructions will result in a mark of 0.

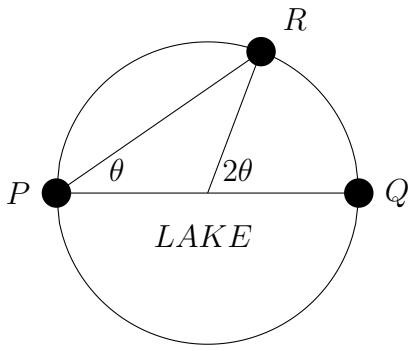
Techniques from this course must be used to solve the questions, not more advanced techniques. For example L'Hopital's Rule for solving limits is not permitted.

The assignment covers sections 4.7, 4.9, 5.1–5.4 in the textbook.

[9] 1. A rectangular storage container is to hold 10 cubic metres. Its length is to be twice as long as the width. The top costs \$4 per square metre and the bottom and sides each cost \$3 per square metre. Determine the dimension which minimize the cost of creating the container. Justify your answer.

[8] 2. Determine the point on the parabola $y = x^2 - 1$ which is closest to the point $P(7, -3)$. Justify your answer. (Hint: If set up correctly, there should be one fairly obvious critical number in the calculation.)

[9] 3. You are at point P on the side of a circular lake of radius 12 metres. You need to get to the point Q directly across the lake. You can either walk clockwise around the lake, swim straight across, or swim at an angle θ to a point R on the shore and finish the trip by walking. You walk 2 m/s and swim 1 m/s. Which path minimizes the amount of time to get from P to Q? See picture below. Justify your answer.



[7] 4. On planet Harland, the acceleration due to gravity is -20 metres per second squared. A rock is thrown from a height of 7.5 metres above the surface with an initial upward speed of of 10 metres per second. Assume no other forces acting on the ball except for gravity. Answer the following questions. You are not permitted to use any formulas from physics.

(a) Determine the maximum height that the ball will reach.

(b) Determine the time it takes for the ball to return to the surface.

[7] 5. If $f''(x) = 3\sin x + 12x^2 + e^x, f(0) = 3,\ f(\pi) = 0$. Determine the function $f(x)$.

[4] 6. Evaluate the following integral by interpreting it in terms of areas. Do not use the fundamental theorem of calculus.

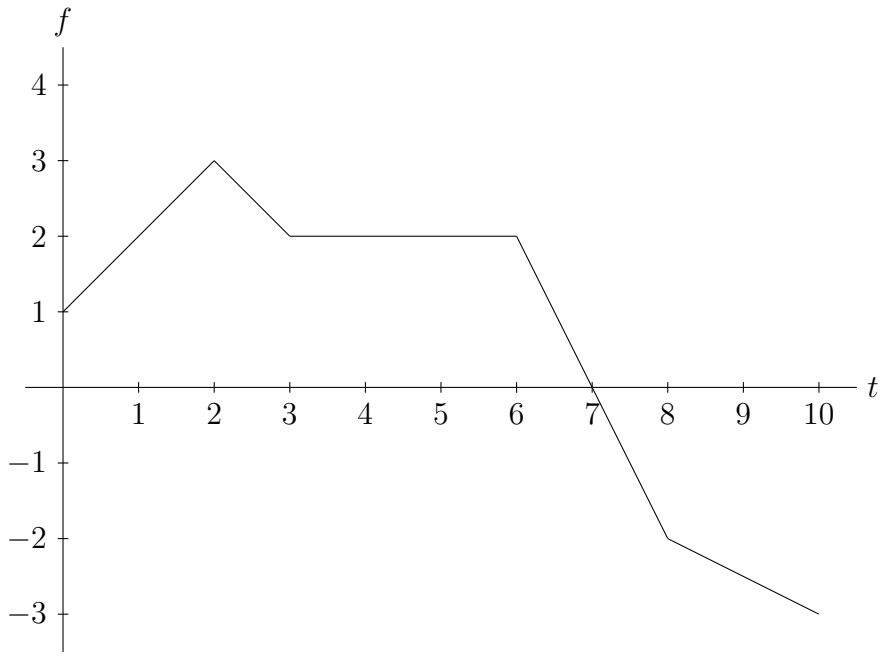
$$\int_{-5}^{5} \left(5 - \sqrt{25 - x^2}\right)\, dx$$

7. Suppose $g(x) = \displaystyle\int_0^x f(t)\, dt$ defined on $[0,\ 10]$ where f is the function sketched below.

[5] (a) Compute $g(0)$, $g(2)$, $g(4)$, $g(6)$ and $g(8)$.

[2] (b) Determine on which interval that g is increasing.

[2] (c) What is the maximum value of g. Explain your answer.



8. Compute the derivatives of

[3] (a) $f(x) = \displaystyle\int_4^{\tan x} \sqrt{1 + t^{10}}\, dt.$

[4] (b) $f(x) = \displaystyle\int_{x^2}^{x^3} e^{t^2}\, dt.$

9. Evaluate the integrals

[4] (a) $\displaystyle\int \left(x^{e^2} + \frac{(2x+3)^2}{x}\right) dx.$

[4] (b) $\displaystyle\int_{\pi/4}^{\pi/2} \csc x \cot x\, dx.$

[7] 10. Compute the area bounded between the curve $y = 4x^3 + 12x^2 - 16x$ and the x–axis. Include a rough sketch of the region.

This assignment is out of 75 points.