DISTANCE EDUCATION
MATH 1500
WINTER TERM 2016: D01/D02

Assignment 1

Sections 1.1, 1.3, 1.5, 1.6, 2.1, 2.2, 2.3.

Total Marks: 60

Due Date: **Jan 23, 2016**.

SHOW ALL WORK to get full marks. Leave answers as exact answers. For example, leave it as $\frac{1}{7}$ as opposed to 0.142857.

1. For the following functions, simplify $\dfrac{F(a+h)-F(a)}{h}$ as much as possible.

[4] (a) $F(x) = 2x^2 - 1$

[4] (b) $F(x) = \dfrac{2}{\sqrt{x+1}}$

2. For the function f defined by $f(x) = -x^2 - 4x - 1$

[2] (a) Put the function in the form $f(x) = -(x-h)^2 + k$.

[3] (b) Find an interval (as large as possible) such that f is one–to–one.

[2] (c) Find the inverse of f on the interval from part (b).

[1] (d) State the domain and range of f using the restriction from part (b).

[2] (e) State the domain and range of f^{-1} using the restriction from part (b).

3. Solve the following equations. If there are any logarithms in the final answer, they should be the natural logarithm.

[4] (a) $\log_3(x) + \log_3(x-8) = 2$

[5] (b) $\dfrac{3^{x-1}}{2^{x-7}} = 1$

[3] (c) $e^{2x} - 5e^x - 6 = 0$

[4] 4. (a) Sketch the graph

$$f(x) = \begin{cases} -(x+1)^2 & x < -1 \\ 4 & x = -1 \\ x+3 & -1 < x < 2 \\ \sqrt{x-2} & x \geqslant 2 \end{cases}$$

[7] (b) Find the following limits if they exists. If they don't exist, explain why.

(a) $\lim_{x \to 1^-} f(x)$

(b) $\lim_{x \to 1^+} f(x)$

(c) $\lim_{x \to 1} f(x)$

(d) $\lim_{x \to 0} f(x)$

(e) $\lim_{x \to 2^-} f(x)$

(f) $\lim_{x \to 2^+} f(x)$

(g) $\lim_{x \to 2} f(x)$

5. Find the following limits if they exist. If they don't exist, determine whether the limit is ∞, $-\infty$ or neither.

[1] (a) $\lim_{x \to 1} \dfrac{e^{x-1} + \ln(x)}{x+1}$

[3] (b) $\lim_{x \to 2} \dfrac{x^3 - 3x - 2}{x^2 - 4}$

[4] (c) $\lim_{x \to 3} \dfrac{x - \sqrt{5x-6}}{x^2 + x - 12}$

[4] (d) $\lim_{x \to 3} \dfrac{e^{x^2}}{x - 3}$

[4] (e) $\lim_{x \to 6} \dfrac{|x-6|}{x^2 - 7x + 6}$

[3] (f) $\lim_{x \to 0^+} (\sin^2 x) \sin\left(\dfrac{1}{x^2}\right)$.