DISTANCE EDUCATION
MATH 1500
WINTER TERM 2016: D01/D02

Assignment 2

Sections 2.5, 2.6, 2.7, 2.8, 3.1, 3.2.

Total Marks: 60

Due Date: **Feb 6, 2016**.

SHOW ALL WORK to get full marks. Leave answers as exact answers. For example, leave it as $\frac{1}{7}$ as opposed to 0.142857. Word problems should be concluded with a sentence and include units. When calculating a derivative, use the definition of derivative ONLY for Q5.

[7] 1. Use limits to calculate a and b such that the following function is continuous for all real numbers x.

$$f(x) = \begin{cases} \sin\left(\dfrac{3\pi}{2}x\right) + ax + b & x < -1 \\ 3 & x = -1 \\ (a+b)x^2 + \cos(x+1) & x > -1 \end{cases}$$

[4] 2. Show that

$$\ln(x+2) = \frac{e^{2x}}{2} - 4x$$

has a solution on the interval $(-1, 0)$. [Note: $\ln(2) \approx .6931$]. Justify your answer.

3. Evaluate the following limits:

[4] (a) $\lim_{x \to -\infty} \left(\dfrac{e^x}{2} + \dfrac{2 - x^2 - x^3}{x^3 + 7} \right)$.

[4] (b) $\lim_{x \to \infty} \dfrac{\sqrt{16x^2 - x}}{x - 1}$.

[5] (c) $\lim_{x \to -\infty} \left(x^3 + \sqrt{x^6 + x^3} \right)$.

[6] 4. Use limits to determine all horizontal and vertical asymptotes for the function

$$y = f(x) = \frac{\sqrt{4x^4 - 1}}{x^2 + x - 6}.$$

[7] 5. Use the definition of the derivative to calculate the derivative of the function

$f(x) = \dfrac{1}{\sqrt{2x}}$.

6. Calculate the derivative of the following functions. Do not simplify your answers.

[3] (a) $f(x) = \sqrt[3]{x} + \dfrac{2}{x^3} + \dfrac{1}{\sqrt[5]{x^2}}$

[4] (b) $f(x) = \dfrac{\pi + xe^x}{e^x - x^2 - x}$

7. Lilly shoots a free throw at her basketball game, its height (in feet) is given by the formula $h(t) = -3t^2 + 15t$ where t is in seconds.

[1] (a) Find the velocity at any time t seconds.

[2] (b) After how many seconds will the basketball stop rising and begin to fall?

[2] (c) After the ball is released from her hand, at what time and with what velocity does the basketball hit the ground.

[5] 8. Calculate an equation of the tangent line to $f(x) = (x^3 + x)e^x$ at $x = 1$.

[6] 9. Let $f(x) = |x|$,

(a) Prove that $f(x)$ is not differentiable at $x = 0$.

(b) Graph $f'(x)$.