DISTANCE EDUCATION
MATH 1500
WINTER TERM 2016: D01/D02

Assignment 3
Sections 3.3, 3.4, 3.5, 3.6, 3.9.
Total Marks: 60
Due Date: **Feb 20, 2016**.

SHOW ALL WORK to get full marks. Word problems should have sentence answers with units.

[17] 1. Differentiate the following functions. DO NOT SIMPLIFY.

(a) $y = \dfrac{x^x}{e^{\tan x} + \cot x}$.

(b) $y = \sec\left(\sqrt{\cos(\pi x - 1)}\right)$

(c) $y = (\sin(x) + e^x \sqrt[3]{x})^3$

(d) $f(t) = \ln|(\pi^2 - t^2 - t^5)|$

(e) $y = \log_5(\csc 3x - \ln \pi^2 + x)$

[6] 2. Evaluate the following limits

(a) $\displaystyle\lim_{\theta \to 0} \dfrac{3 \sin \theta}{\theta + 2 \tan \theta}$

(b) $\displaystyle\lim_{t \to -1} \dfrac{t \, \sin(t + 1)}{t^3 + 3t^2 + 2t}$

[5] 3. Let c and k be any two distinct real numbers. Prove that if f and g are both differentiable, then

$$(c\,f(x) + k\,g(x))' = c\,f'(x) + k\,g'(x).$$

[3] 4. Let $h(x) = 2^{\,b\tan(x)+1}$. For what value of b is $h'(\pi) = 1$.

[7] 5. Use implicit differentiation to find the equation of the tangent line to the curve

$$\sqrt{x} + \sin(xy) - e^{y-\pi} = 0$$

at the point $(1, \pi)$.

[8] 6. Use logarithmic differentiation to find the derivative of

$$y = \dfrac{\sqrt[3]{x}(x - 1)^5 \ln(x)}{e^x}$$

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[7] 7. (Leaky Tank) Water is pouring into a leaky tank at a rate of 10 m^3/h. The tank is a cone with vertex down, 9 m in depth and 6 m in diameter at the top. The surface of water in the tank is rising at a rate of $\frac{1}{5}$ m/h when the depth is 6 m. How fast is the water leaking out at that time?

[7] 8. (Tracking a balloon) A balloon is released from a point A and rises vertically with constant vertical speed of 5 m/s. Point B is level with point A and is 100 m distance from point A. How fast is the angle of elevation of the balloon at B changing when the balloon is 200 m above A?