DISTANCE EDUCATION
MATH 1500
WINTER TERM 2016: D01/D02

Assignment 4
Sections 4.1, 4.2, 4.3, 4.5.
Total Marks: 60
Due Date: **Mar 6, 2016**.

SHOW ALL WORK to get full marks.

1. Find the critical numbers of the function.

[3] (a) $f(x) = 15x^3 + 12x^2 + 3x$.

[3] (b) $g(t) = |2t - 6|$

[5] 2. Find the absolute maximum and absolute minimum values of f on the given interval. Justify your answer.

$$f(x) = 3x^{2/3} - 2x, \quad [-1, 1].$$

[6] 3. Verify that the following function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

$$f(x) = 2x^3 + x - 3, \quad [-1, 0].$$

4. Consider $f(x) = \dfrac{1}{x^3 - 4x}$. Note then that

$$f'(x) = \frac{4 - 3x^2}{x^2(x^2 - 4)^2} \qquad \text{and} \qquad f''(x) = \frac{4(3x^4 - 6x^2 + 8)}{x^3(x^2 - 4)^3}.$$

[2] (a) Find the domain of the function.

[2] (b) Find the x and y coordinate(s) of the critical numbers.

[4] (c) Find all vertical asymptote(s) of $f(x)$. (Show all your work).

[2] (d) Find the open intervals where the function is increasing.

[4] (e) Find the open intervals where the function is decreasing.

[2] (f) Find the x and y coordinate(s) of any local(relative) extrema using the first derivative test.

[4] (g) Find the x and y coordinate(s) of any local(relative) extrema using the second derivative test. [Hint: $3x^4 - 6x^2 + 8 \geqslant 5$]

5. Consider the curve given by the function

$$f(x) = \frac{x^2 - x}{(x+1)(x-2)}.$$

Then

$$f'(x) = \frac{2 - 4x}{(x-2)^2(x+1)^2} \qquad \text{and} \qquad f''(x) = \frac{12(x^2 - x + 1)}{(x-2)^3(x+1)^3}.$$

[2] (a) Determine the domain of $f(x)$

[2] (b) Find the x-intercepts.

[4] (c) Find all vertical asymptote(s) of $f(x)$. (Show all your work).

[2] (d) Find all horizontal asymptote(s) of $f(x)$. (Show all your work).

[2] (e) Find all critical points of $f(x)$ (that is, all critical numbers, together with their y values).

[2] (f) Find the open intervals where $f(x)$ is increasing and the open intervals where $f(x)$ is decreasing.

[2] (g) Find the x and y coordinates of all local maxima and/or local minima.

[2] (h) Determine the open intervals upon which $f(x)$ is concave up and the open intervals where $f(x)$ is concave down. [Hint: $x^2 - x + 1 \geqslant \frac{3}{4}$]

[1] (i) Find the x and y coordinates of all inflection point(s).

[4] (j) Use the above information to give a neat sketch of the graph $y = f(x)$.