

Math 1500 D01 Fall 2017
Written Assignment #1 (amended Oct 1)
Due Tuesday, Oct 3, 11:55 PM

Instructions

- Each assignment must be submitted online by the deadline in UM Learn as a single PDF file. It may be scanned from your own (neatly organized, legible) handwritten work composed originally, by yourself in typesetting software.
- Late assignments will not be accepted.
- Failure to follow instructions will be penalized up to and including receiving 0 on an assignment.
- Show all work for full marks.
- Leave numerical answers as (simplified) exact-value expressions; for example $\frac{1}{7}$ should be left as a fraction and not approximated by 0.142857; similarly for $\sqrt{3}$ and $17\pi + 2$. You will be penalized wherever “=” is used to equate numbers that are only approximately equal.
- Simplify answers—within reason. For example fractions should be in lowest terms, 0 should not appear in a sum of more than one term; 1 should not appear in a product of more than one factor. $5 + 7$ simplifies to 12 and $(x + 1)^2 - (x - 3)^2$ simplifies to $8x - 8$ or $8(x - 1)$.
- Calculators are not permitted in assignments; all work should be carried out by hand.
- You are expected to use techniques from this course to solve problems in assignments. For example, no marks will be given for derivation of limits using L'Hopital's Rule.
- “Guess and check”, except where clearly the expected approach, will be penalized and possibly given no credit.

This assignment covers §1.1, 1.3, 1.4, 1.5, 2.2, 2.3, 2.5, 2.6

1. For each of the following functions $f(x)$, simplify the difference quotient $\frac{f(a+h) - f(a)}{h}$ as far as possible. In (a), (b) it is expected that h will not appear as a factor in either the numerator or denominator of the resulting expression.

[3] (a) $f(x) = x^3 + x$

[3] (b) $2\sqrt{x+3}$

[3] (c) $2^x + 1$

[5] 2. Find the domain of $p(x) = \frac{x^2 - 3x}{x^2 - 5x + 6} + \sqrt{6 - x}$. Report your answer in interval form.

3. Define a function by $f(x) = x^2 - 8x + 5$.

[2] (a) Rewrite this function in the form $f(x) = (x - h)^2 + k$.

[2] (b) Sketch a graph of this function by comparing $y = f(x)$ to $y = x^2$.

[2] (c) Find an interval, as large as possible, on which f is a one-to-one function.

- [2] (d) Write down the inverse of f on the interval you found in (c) above.
4. Solve each of the following equations for x . In your answers, do not use any logarithms except for natural logarithm.
- [4] (a) $\log_3(5x + 2) - \log_9(2x + 1) = -1$
- [4] (b) $2^{x+1} = \sqrt{3^{2x-1}}$
- [4] (c) $e^{2x+2} + 6 = 5e^{x+1}$

5. Let $f(x) = \begin{cases} x + 1 & x \leq -1 \\ x^2 & -1 < x < 0 \\ 3x & 0 < x < 1 \\ 5 & x = 1 \\ 2 - x & x > 1 \end{cases}$

- [3] (a) Roughly sketch a graph of $y = f(x)$, being sure to indicate whether points are included or excluded from the curve at ends of subdomain intervals.
- [6] (b) Find $\lim_{x \rightarrow a} f(x)$, $\lim_{x \rightarrow a^-} f(x)$, $\lim_{x \rightarrow a^+} f(x)$ for each of $a = -1, 1, 2$. If any don't exist, explain.
- [3] (c) At which of points $-1, 0, 1$ is $f(x)$ discontinuous? Explain your reasoning for each.
6. Calculate any limits that exist, including any which are infinite limits, and reporting "DNE" otherwise.

- [2] (a) $\lim_{x \rightarrow \frac{3\pi}{4}} \frac{\sin x}{x}$
- [2] (b) $\lim_{x \rightarrow \pi^-} \frac{x}{\sin x}$
- [4] (c) $\lim_{x \rightarrow 2} \frac{x^3 + 2x^2 - 5x - 6}{x^2 - 4}$ (HINT: factor)
- [4] (d) $\lim_{x \rightarrow 0} \frac{x^2 + 2x}{\sqrt{x^2 + 9} - \sqrt{x + 9}}$ (HINT: rationalize)
- [4] (e) $\lim_{t \rightarrow -5} \frac{|2t + 10|}{t^2 - t - 30}$
- [4] (f) $\lim_{x \rightarrow 3^+} \frac{2x^2 - 18}{5(x - 3)^2}$
- [4] (g) $\lim_{x \rightarrow 0^+} \frac{1}{x - \sqrt{x^2 + x}}$

- [5] 7. For which values of a and b is the following function continuous everywhere? Show all the steps of your reasoning for full marks; an unjustified correct answer is worth only 1 mark.

$$g(x) = \begin{cases} 2ax + b & x < 1 \\ a^2 + b^2 & x = 1 \\ 7x^3 - (a + b)x + 1 & x > 1 \end{cases}$$

- [4] 8. Use a result from this course to prove that $g(x) = x^3 - 11x - 21$ has a zero somewhere in the interval $(4, 5)$.

Total marks on this assignment: 80