Math 1500 D01 Fall 2017
Written Assignment #2
Due Monday, Oct 30, 5:55 PM

Instructions

- Each assignment must be submitted online by the deadline in UM Learn as a single PDF file. It may be scanned from your own (neatly organized, legible) handwritten work or composed originally, by yourself in typesetting software.
- Late assignments will not be accepted.
- Failure to follow instructions will be penalized up to and including receiving 0 on an assignment.
- Show all work for full marks.
- Leave numerical answers as (simplified) exact-value expressions; for example $\frac{1}{7}$ should be left as a fraction and not approximated by 0.142857; similarly for $\sqrt{3}$ and $17\pi + 2$. You will be penalized wherever "=" is used to equate numbers that are only approximately equal.
- Simplify answers—within reason. For example fractions should be in lowest terms, 0 should not appear in a sum of more than one term; 1 should not appear in a product of more than one factor. $5 + 7$ simplifies to 12 and $(x + 1)^2 - (x - 3)^2$ simplifies to $8x - 8$ or $8(x - 1)$.
- Calculators are not permitted in assignments; all work should be carried out by hand.
- You are expected to use techniques from this course to solve problems in assignments. For example, no marks will be given for derivation of limits using L'Hopital's Rule.
- "Guess and check", except where clearly the expected approach, will be penalized and possibly given no credit.

This assignment covers the following sections of Stewart's text: §2.7, 2.8, 3.1-3.5, 3.9

1. Find the derivative $f'(x)$ "from first principles" (which means using the definition of derivative), where

$$f(x) = \sqrt{2x^2 + 3}$$

[7] Solutions using the limit laws will receive very little credit.

2. Obtain the following derivatives using any method. Leave answers unsimplified.

[4] (a) $f(x) = x^{2017} + \sqrt[3]{x^{2017}} + \dfrac{3}{x^{2017}}$

[4] (b) $g(x) = \dfrac{x \sin x - e^x}{x^2 - 1}$

[4] (c) $k(t) = \sqrt{3e^{\sin 2t}}$

[4] (d) $p(\theta) = \csc(2^\theta + \tan\theta)$

3. Use the two basic limits involving $\sin$ and $\cos$ to determine the value of the following limits, showing your reasoning.

[4] (a) $\displaystyle\lim_{x\to 1} \frac{2\sin(t-1)}{t^2 + 3t - 4}$

[4] (b) $\displaystyle\lim_{y\to 0} \frac{\cos^2 y - 1}{\sqrt{y+1} - 1}$

4. An arrow is shot skyward at time $t = 0$, and its height (in meters) at time t (in seconds) during its flight is given by the formula

$$h(t) = 70t - 4.9t^2$$

   [2] (a) Find an expression for the velocity of the arrow at any time t during its flight (treating "upwards" as the positive direction).

   [4] (b) At what time during the flight does the arrow's motion come momentarily to a standstill (i.e. when does it stop rising and start falling)?

   [4] (c) What acceleration is the arrow undergoing after 5 seconds?

   [4] (d) At what velocity will the arrow strike the ground?

[5] 5. Find an equation for the line tangent to the graph of $f(x) = \dfrac{x^2}{x^3+1}$ when $x = 3$

6. Given that $f(1) = 5$, $f'(1) = 2$, $g(3) = 1$ and $g'(3) = 2$:

   [5] Find the value of $F'(3)$, where $F(x) = xf\left(\dfrac{x}{3}\right) + \dfrac{x^2}{g(x)} + f(g(x))$

[6] 7. Use implicit differentiation to find an equation for the line tangent to $x^2 + xy - y^3 = 8e^{x+y}$ at the point $(2, -2)$

[8] 8. A lighthouse on a rock 100 m from a straight shoreline has a rotating spotlight whose beam makes a complete rotation once per minute. At what speed, in meters per second, is the light beam travelling along the shore when it shines on a dock 500 m from the closest point on the shore to the rock? (Leave your answer as a simplified but unevaluated formula, not a decimal number.)

[10] 9. Water is pouring at a rate of 1 cubic meter per second into a holding tank in the shape of an inverted cone with height 8 m and radius 6 m. Find the rate at which the surface area (meaning the circular top surface of the water) is increasing when the water is 4 m deep. (HINT: You are interested in the height, radius, surface area and volume of the water in the tank, all as functions of time. Either eliminate some variables to clarivy relations between quantities, or solve a chain of two related rates problems)

Total marks on this assignment: 80.