MATH 1500 D01 Winter 2017 Assignment 1

SHOW ALL WORK to get full marks. Leave answers as exact answers. For example, leave it as fractions such as 1/7 as opposed to decimals such as 0.142857 or $\ln 4$ or e^{15}. Word problems should have sentence answers with units. Fractions should be lowest terms.

Calculators are not permitted. Assignments using a calculator will not be graded.

All assignments must be handed in on UMLearn as **one PDF file**. Late assignments will not be accepted. Failure to follow the instructions will result in a mark of 0.

Techniques from this course must be used to solve the questions, not more advanced techniques. For example L'Hopital's Rule for solving limits is not permitted.

This assignment covers sections 1.1, 1.3, 1.4, 1.5, 2.2, 2.3, 2.5 and 2.6 in the textbook.

1. For the following functions, simplify $\dfrac{f(a+h)-f(a)}{h}$ as much as possible. (At minimum, neither answer should have a **factor** of h when you are done. Further note that this question is not asking for a derivative nor a limit.)

[4] (a) $f(x) = x^2 - 6x + 7$

[4] (b) $f(x) = \dfrac{1}{\sqrt{3x-2}}$

[4] 2. Compute the domain of $\dfrac{\sqrt{x^2-2x-3}}{2x+3}$. Leave your answer in interval notation.

3. For the function f defined by $f(x) = x^2 + 4x + 5$:

[2] (a) Convert the function to the form $f(x) = (x-h)^2 + k$.

[2] (b) Find an interval such that f is one–to–one.

[2] (c) Find the inverse of f on the interval from part (b).

4. Solve the following equations. If there are any logarithms in the final answer, they should be the natural logarithm.

[4] (a) $\log_5(3-2x) + \log_5(24-x) = 3$

[5] (b) $3^{x-1} = 4\left(5^{2x+3}\right)$

[3] (c) $e^{2x} - 5e^x - 14 = 0$

[3] 5. (a) Sketch the graph

$$f(x) = \begin{cases} -2x-7 & x < -2 \\ 2 & x = -2 \\ (x+2)^2 - 3 & -2 < x \leqslant 1 \\ 3-x & x > 1 \end{cases}$$

[4] (b) Find the following limits if they exists. If they don't exist, explain why

$$\lim_{x\to -2^-} f(x) \quad \lim_{x\to -2^+} f(x) \quad \lim_{x\to -2} f(x) \quad \lim_{x\to 0} f(x) \quad \lim_{x\to 1^-} f(x) \quad \lim_{x\to 1^+} f(x) \quad \lim_{x\to 1} f(x)$$

[3] (c) Is the function continuous at the values $x = -2, x = 0$ and $x = 1$. Explain your answer.

6. Find the following limits if they exist. If they don't exist, determine whether the limit is ∞, $-\infty$ or neither.

[1] (a) $\lim_{x \to 1^+} \dfrac{x^2 + 4x}{2x^2 + 3x + 6}$

[3] (b) $\lim_{x \to 1} \dfrac{x^3 - 3x^2 - 4x + 6}{x^2 - 1}$ (Hint: If $x = 1$ makes a polynomial equal to 0 then what must be a factor?)

[4] (c) $\lim_{x \to 3} \dfrac{x^2 + 2x - 15}{\sqrt{x - 2} - \sqrt{4 - x}}$

[4] (d) $\lim_{x \to 3^-} \dfrac{x^2 - 7x + 10}{x^2 - 6x + 9}$

[4] (e) $\lim_{x \to 2} \dfrac{x^2 + 2x - 8}{|6 - 3x|}$

[3] (f) $\lim_{x \to 0^+} x \sin\left(\dfrac{2}{x^2}\right)$

[4] (g) $\lim_{x \to -\infty} \dfrac{3x + 5}{6x + \sqrt{x^2 - 2x}}$

[3] (h) $\lim_{x \to \infty} \dfrac{1}{x - \sqrt{x^2 + 6x}}$

[5] 7. Use limits to calculate a and b such that the following function is continuous for **all real numbers** x. Justify your answer.

$$f(x) = \begin{cases} x^2 + ax + b & x < 2 \\ 5 & x = 2 \\ \ln(x - 1) + (b - a)x - 9 & x > 2 \end{cases}$$

[4] 8. Show that $2x^4 = 4x - 1$ has a solution on the interval $(1, 2)$. Justify your answer.

This assignment is out of 75 points.