MATH 1500 D01 Winter 2017 Assignment 2

SHOW ALL WORK to get full marks. Leave answers as exact answers. For example, leave it as fractions such as 1/7 as opposed to decimals such as 0.142857 or $\ln 4$ or e^{15}. Word problems should have sentence answers with units. Fractions should be lowest terms.

Calculators are not permitted. Assignments using a calculator will not be graded.

All assignments must be handed in on UMLearn as **one PDF file**. Late assignments will not be accepted. Failure to follow the instructions will result in a mark of 0.

Techniques from this course must be used to solve the questions, not more advanced techniques. For example L'Hopital's Rule for solving limits is not permitted.

This assignment covers sections from 2.7, 2.8, 3.1–3.5, 3.9.

Use the definition of derivative only for Q1. Do not use the definition in any other questions.

[7] 1. Use the definition of the derivative to calculate the derivative of the function $f(x) = \dfrac{1}{\sqrt{5-x^2}}$.

2. Calculate the derivative of the following functions. Do not simplify your answers.

[3] (a) $f(x) = \sqrt[4]{x} + \dfrac{2}{x^{2017}} - \dfrac{2}{\sqrt{x^5}}$

[4] (b) $f(x) = \dfrac{5 - x\sin x}{x + e^x}$

[5] (c) $f(x) = e^{e^{3\cot x}}$

[5] (d) $f(x) = \cos(\sqrt{e^{5x} + \tan x})$

3. Evaluate the following limits. Show all steps.

[4] (a) $\displaystyle\lim_{x\to 0} \frac{3\sin 4x}{\sin 7x}$

[4] (b) $\displaystyle\lim_{t\to 3} \frac{4\sin(t-3)}{(t^2 - 8t + 15)}$

4. If a rock is thrown upwards with a velocity of 20 metres per second, its height, in metres, is given by

$$H(t) = -4.9t^2 + 20t.$$

(Calculus method and not physics methods must be used)

[1] (a) Find the velocity at any time t seconds.

[3] (b) After how many seconds will the rock stop rising and begin to fall?

[4] (c) After what time, and with what velocity will the rock hit the ground?

[5] 5. Calculate an equation of the tangent line to $f(x) = (x^4 + 4x - 5)e^x$ when $x = 0$.

[4] 6. If $f(2) = 4$ and $f'(2) = -3$, calculate $g'(2)$ if $g(x) = x^2 f(x)$.

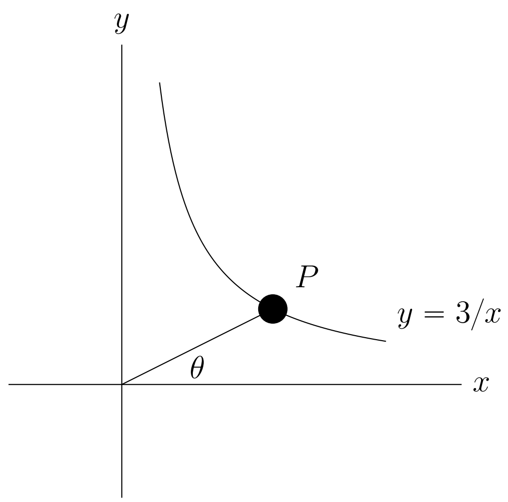
[8] 7. Use implicit differentiation to find the equation of the tangent line to the curve

$$x^3 + \sin xy + y^3 = e^{2y} - 1$$

at the point $(1, 0)$.

[9] 8. A streetlight, 5 metres tall, shines on the ground. A person 2 metres tall walks towards the streetlight at a speed of 4/5 metres per second. How fast is the length of their shadow changing when they are 4 meters from the streetlight

[9] 9. A particle P is moving along the curve $y = \dfrac{3}{x}$ so that its x coordinate (in meters) is increasing at a rate of 4 m/s. The line segment between the origin (0, 0) and P forms an angle θ between the line segment and the positive x-axis. See the figure below



Compute the rate of change of θ with respect to time when the particle passes through $x = \sqrt{3}$. (Hint: You may use $\sec^2\theta = 1 + \tan^2\theta$ at some point, although it is not necessary.)

This assignment is out of 75 points.