MATH 1700: Assignment 1

Due: 11:59 pm (Central time = time in Winnipeg), Monday, 6 February 2017

Paper with solutions has to be submitted as one PDF file

(Late assignments will not be accepted)

Question:	1	2	3	4	5	6	7	8	Total
Points:	25	10	5	5	5	5	5	10	70
Score:									

1. Find the limit or explain why it does not exist. You can use any method. If you are using l'Hospital's Rule explain why it applies.
   (a) (5 points) $\lim_{x\to 0} \dfrac{\sqrt{5+x}-\sqrt{5-x}}{x}$
   (b) (5 points) $\lim_{x\to \pi/2} \dfrac{\tan x}{x}$
   (c) (5 points) $\lim_{x\to -\infty} xe^x$
   (d) (5 points) $\lim_{y\to \infty} y\sin(1/y)$
   (e) (5 points) $\lim_{w\to 0}(1+w)^{\cot w}$
2. Consider the following parametric curve:
$$x = \sqrt{-t^2+2t+3}, \quad y = \frac{1}{-t^2+2t+3}$$
   (a) (1 point) Eliminate the parameter to find a Cartesian equation of the curve (don't worry about the domain in this part).
   (b) (4 points) Find all values of x for which your Cartesian equation represents this parametric curve (hint: range of x? is y defined for all those x?).
   (c) (5 points) Sketch the parametric curve and indicate with arrows the direction in which the curve is traced as the parameter increases. If the direction changes, indicate the value of the parameter t for which this change occurs.
3. (5 points) Assume that y is defined parametrically as a function of x, and find dy/dx (simplify your answer as much as possible):
$$x = \frac{u}{u-1}, \quad y = \frac{u^2}{u^2-1}.$$

4. (5 points) Find an equation of the tangent line to the curve

$$x = \sqrt{t-1}, \quad y = (t^3+1)^{3/2}$$

at the point $(1, 27)$.

5. (5 points) Find the distance between the points with polar coordinates $(1, \pi/4)$ and $(1, \pi)$.

6. (5 points) Sketch the region in the plane consisting of points whole polar coordinates satisfy the following equations:

$$1 \leq r, \quad r\sin\theta < 1, \quad \pi/4 \leq \theta < \pi/2.$$

7. (5 points) Find an equation of the tangent line to the polar curve $r = 10\cos(2\theta)$ at the point with polar coordinates $(5, \pi/6)$.

8. Use the properties of integrals to verify the following inequalities without evaluating the integrals (make sure to explain your reasoning):

   (a) (5 points) $\displaystyle\int_0^1 (1+x^2)^{1/3}\,dx \leq \int_0^1 (1+x^2)^5\,dx$

   (b) (5 points) $\displaystyle\int_{-\pi/3}^{\pi/3} \left(1 + 2^{10}\cos^{10} x\right)\,dx \geq \frac{4\pi}{3}$