Assignment 1 Solutions

1. We have reduced the margin of error by a factor of $\frac{5}{2} = 2.5$, so we require $(2.5)^2 = 6.25$ times the original sample size. That is, we require a sample of size $50(6.25) = 312.5 \approx 313$.

2. Since this is a one-sided test with $\alpha = 0.03$, we must find the value z^* such that $P(Z > z^*) = 0.03$, or, equivalently, $P(Z < z^*) = 0.97$. From Table 2, we find that $z^* = 1.88$. We will reject $\mathrm{H_0}$ if $Z \geq z^* = 1.88$.

3. The test statistic is $z = \dfrac{\overline{x} - \mu_0}{\sigma/\sqrt{n}} = \dfrac{112 - 110}{15/\sqrt{30}} = 0.73$.

   Since $z = 0.73 < z^* = 1.88$, we fail to reject $\mathrm{H_0}$.

4. Since this is a two-sided test with $\alpha = 0.05$, we must find the values $-z^*$ and z^* such that $P(Z < -z^*) = 0.025$ and $P(Z > z^*) = 0.025$. From Table 3, we find that $z^* = 1.96$.

   We will reject $\mathrm{H_0}$ if

   $$Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \leq -z^* \text{ or if } Z = \frac{\overline{X} - \mu_0}{\sigma/\sqrt{n}} \geq z^*$$

   $$\Rightarrow \overline{X} \leq \mu_0 - z^* \frac{\sigma}{\sqrt{n}} = 75 - 1.96\frac{10}{\sqrt{15}} = 69.9393$$

   $$\text{or } \overline{X} \geq \mu_0 + z^* \frac{\sigma}{\sqrt{n}} = 75 + 1.96\frac{10}{\sqrt{15}} = 80.0607$$

5. Power $= P(\text{Reject } \mathrm{H_0} \mid \mathrm{H_a} \text{ true})$

   $$= P(\overline{X} \leq 69.9393 | \mu = 82) + P(\overline{X} \geq 80.0607 | \mu = 82)$$

   $$= P\left(Z \leq \frac{69.9393 - 82}{10/\sqrt{15}}\right) + P\left(Z \geq \frac{80.0607 - 82}{10/\sqrt{15}}\right)$$

   $$= P(Z \leq -4.67) + P(Z \geq -0.75) = 0 + (1 - 0.2266) = 0.7734$$

6. When we decrease the level of significance (the probability of a type I error), this results in an increase in the probability of a type II error, which thus decreases power.

7. When we increase the sample size, the power increases.

8. In Question 5, we calculated the power to detect a true mean of 82, with $\mu_0 = 75$. That is, we were calculating the power to detect a true mean $82 - 75 = 7$ beats per minute away from $\mu_0 = 75$. Now, the alternative mean is 85 beats per minute, so the size of the difference we want to detect is now $85 - 75 = 10$. When we increase the size of the difference we want to detect, the power increases.

9. Since we don't know the value of the population standard deviation σ, we will be using the t distribution to conduct our inference procedures.

   A 90% confidence interval for the true mean amount of detergent per box is
   $\overline{x} \pm t^* \frac{s}{\sqrt{n}} = 38.69 \pm 1.833 \frac{0.66}{\sqrt{10}} = 38.69 \pm 0.38 = (38.31, 39.07)$
   where $t^* = 1.833$ is the upper 0.05 critical value of the t distribution with 9 degrees of freedom.

10. The test statistic is $t = \frac{\overline{x} - \mu_0}{s/\sqrt{n}} = \frac{38.69 - 39}{0.66/\sqrt{10}} = -1.49$.

11. We are testing the hypotheses H_0: $\mu = 39$ vs. H_a: $\mu < 39$. So, the P-value is $P(T(9) \leq -1.49) = P(T(9) \geq 1.49)$. We see from Table 3 that

    $$P(T(9) \geq 1.383) = 0.10 \text{ and } P(T(9) \geq 1.833) = 0.05.$$

    Since $1.383 < 1.49 < 1.833$, the P-value is between 0.05 and 0.10.

12. Since the P-value $< \alpha = 0.10$, we reject H_0.

13. Since this is a left-sided test with $\alpha = 0.10$, we must find the value $-t^*$ such that $P(T(9) < -t^*) = 0.10$. From Table 3, the value of t^* that satisfies $P(T(9) > t^*) = 0.10$ is $t^* = 1.383$. So, $-t^* = -1.383$. We will reject H_0 if $t \leq -1.383$.