Assignment 2 Solutions

1. Since each child involved in the study is receiving only one of the two teaching methods, there is no natural pairing between observations in the two groups. So these are two independent samples. The ratio of the standard deviations

$$\frac{\max s_i}{\min s_i} = \frac{8}{5} = 1.6 < 2,$$

   so we use the pooled independent two-sample t test.

2. The degrees of freedom for a pooled independent two-sample t test is $n_1 + n_2 - 2 = 24$.

3. This is an example of paired data, so we will use the matched pairs t procedures. We are concerned not with the weights prior to beginning the diet and after six weeks on the diet individually, but with the **difference** in before-diet and after-diet weights for each individual.

   In order to conduct a matched pairs t test, we must assume that differences in before-diet and after-diet weights are normally distributed.

   For each person, before-diet weight and after-diet weight are dependent (related).

4. The margin of error is the quantity to the right of the $\pm$ sign (the critical value times the standard error) when computing a confidence interval. There are $n-1 = 19$ degrees of freedom, so for a 95% confidence interval, the critical value $t^* = 2.093$. The standard error is $\frac{s}{\sqrt{n}} = \frac{13}{\sqrt{20}} = 2.9069$. Therefore, the margin of error is $2.093 \times 2.9069 = 6.08$.

5. The test statistic is $t = \frac{\overline{x}_d}{s_d/\sqrt{n}} = \frac{7}{13/\sqrt{20}} = 2.408$. Since we are conducting an upper-tailed test (the hypotheses are H_0: $\mu_d = 0$ vs. H_a: $\mu_d > 0$), the P-value is $P(T(19) \geq 2.408)$. From Table 3, we see that

$$P(T(19) \geq 2.205) = 0.02 \quad \text{and} \quad P(T(19) \geq 2.539) = 0.01$$

   Since $2.205 < 2.408 < 2.539$, the P-value is between 0.01 and 0.02.

6. If the true mean of differences in weights before and after undertaking the diet were zero, the probability of observing a sample mean difference at least as high as 7 pounds would be between 0.01 and 0.02.

7. The ratio of the standard deviations

$$\frac{\max s_i}{\min s_i} = \frac{7}{4} = 1.75 < 2,$$

so we will use the pooled two-sample inference procedures. The estimate of the common variance is

$$s_p^2 = \frac{(n_1-1)s_1^2 + (n_2-1)s_2^2}{n_1+n_2-2} = \frac{13(7)^2 + 17(4)^2}{14+18-2} = 30.3$$

A 99% confidence interval for the difference in true mean ages of Liberals and Conservatives is

$$\begin{aligned} \overline{x}_1 - \overline{x}_2 \pm t^* \sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)} &= 43 - 38 \pm 2.750\sqrt{30.3\left(\frac{1}{14}+\frac{1}{18}\right)} \\ &= 5 \pm 5.39 \\ &= (-0.39, 10.39) \end{aligned}$$

where $t^* = 2.750$ is the upper 0.005 critical value of the t distribution with $n_1 - n_2 - 2 = 30$ degrees of freedom.

8. If we took repeated samples of 14 registered Liberals and 18 registered Conservatives and calculated the interval in a similar manner, 99% of such intervals would contain the difference in the true mean ages of registered Liberals and registered Conservatives.

9. Since this is a two-sided test with $\alpha = 0.01$, we must find the values $-t^*$ and t^* such that $P(T(30) < -t^*) = 0.005$ and $P(T(30) > t^*) = 0.005$. From Table 3, we find that $t^* = 2.750$. We will reject H_0 if $t < -2.750$ or if $t > 2.750$.

10. The test statistic is $t = \dfrac{\overline{x}_1 - \overline{x}_2}{\sqrt{s_p^2\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}} = \dfrac{43-38}{\sqrt{30.3\left(\frac{1}{14}+\frac{1}{18}\right)}} = 2.549.$
Since $-2.750 < t = 2.549 < 2.750$, we fail to reject H_0.

11. Since this is a two-sided test (the hypotheses are H_0: $\mu_1 - \mu_2 = 0$ vs. H_a: $\mu_1 - \mu_2 \neq 0$), and since the level of significance (1%) and the confidence level (99%) add up to 100%, we could have used the interval to conduct the test. Since the value 0 is contained in the 99% confidence interval for $\mu_1 - \mu_2$, we would fail to reject H_0 at the 1% level of significance.

12. The null and alternative hypotheses must be stated in terms of parameters like μ instead of statistics like $\overline{x}$. The test is one-sided, so the answer is H_0: $\mu_{\mathrm{A02}} = \mu_{\mathrm{A01}}$ vs. H_a: $\mu_{\mathrm{A02}} > \mu_{\mathrm{A01}}$.

13. In general, a type I error is committed when we reject H_0 (and conclude that there is sufficient evidence that H_a is true) even though H_0 is true.

In the context of this example, a type I error is committed if we conclude that the true mean midterm score for Section A02 students is higher than the true mean midterm score for Section A01 students even though there is no difference in the true mean midterm scores for Section A02 and A01 students.

14. The ratio of the standard deviations

$$\frac{\max s_i}{\min s_i} = \frac{20.20}{9.27} = 2.18 > 2,$$

so we will use the conservative (unpooled) independent two-sample t test. The degrees of freedom is

$$\min\{n_1 - 1, n_2 - 1\} = \min\{9, 7\} = 7.$$

The test statistic is $t = \dfrac{\overline{x}_1 - \overline{x}_2}{\sqrt{\dfrac{s_1^2}{n_1} + \dfrac{s_2^2}{n_2}}} = \dfrac{71.0 - 64.6}{\sqrt{\dfrac{9.27^2}{8} + \dfrac{20.20^2}{10}}} = 0.891$. Since we are conducting an upper-tailed test, the P-value is $P(T(7) \geq 0.891)$. From Table 3, we see that

$$P(T(7) \geq 0.711) = 0.25 \quad \text{and} \quad P(T(7) \geq 0.896) = 0.20$$

Since $0.711 < 0.891 < 0.896$, the P-value is between 0.20 and 0.25.

15. Since the P-value is greater than 0.05, we fail to reject H_0 and we have insufficient evidence to conclude that the true mean midterm score for Section A02 students is higher than the true mean midterm score for Section A01 students.