$$f(x) = \begin{cases} x^2+3 & \text{if } x \le 2 \\ 3x+1 & \text{if } x > 2 \end{cases}$$

(a) Is $f(x)$ continuous at $x=2$? Justify your answer.

(b) Is $f(x)$ differentiable at $x=2$? Justify your answer.

ANSWER

(a) $f(2) = 2^2+3 = 7$

$\lim_{x\to 2^-} f(x) = \lim_{x\to 2^-} (x^2+3) = 7$

$\lim_{x\to 2^+} f(x) = \lim_{x\to 2^+} (3x+1) = 7$

$f(x)$ is continuous at $x=2$ because

$\lim_{x\to 2} f(x) = f(2)$; $(7=7)$.

(b) $f(x)$ is differentiable at $x=2$ if and only if $f(x)$ is continuous at $x=2$ and $\lim_{x\to 2^-} f'(x) = \lim_{x\to 2^+} f'(x)$.

We know $f(x)$ is continuous at $x=2$ (see (a))

$\lim_{x\to 2^-} f'(x) = \lim_{x\to 2^-} (2x) = 4$

$\lim_{x\to 2^+} f'(x) = \lim_{x\to 2^+} (3) = 3$

$\lim_{x\to 2^-} f'(x) \neq \lim_{x\to 2^+} f'(x)$ $(4 \neq 3)$

$f(x)$ is not differentiable at $x=2$ ($f'(2)$ does not exist.)