Grant's Tutoring

CALCULUS

for MANAGEMENT

Volume 1 of 2

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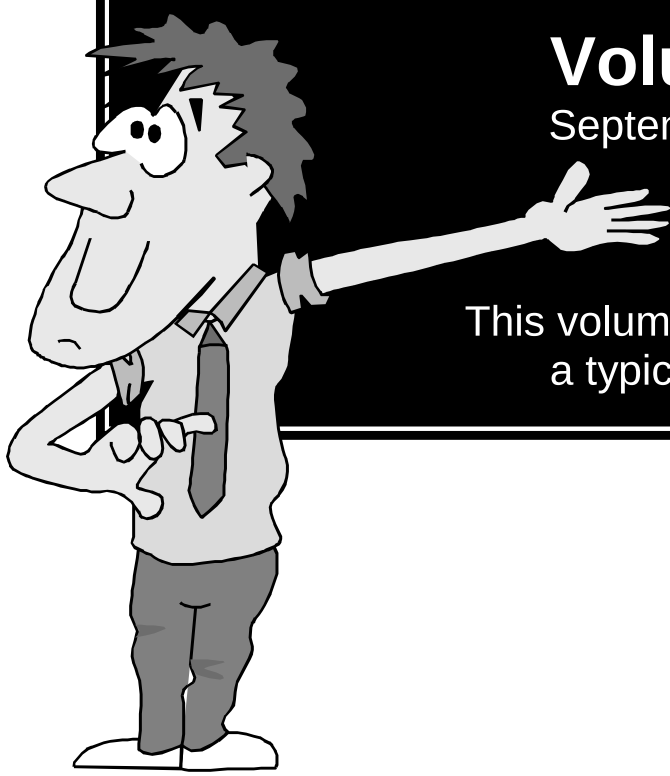


This volume covers the topics on a typical midterm exam.

Learn What You Need to Know

Know What You Need to Learn

While studying this book, why not hear Grant explain it to you? Contact Grant for info about purchasing **Grant's Audio Lectures**. Some concepts make better sense when you hear them explained.

Better still, see Grant explain the key concepts in person. Sign up for **Grant's Weekly Tutoring** or attend **Grant's Exam Prep Seminars.** Text or Grant (204) 489-2884 or go to **www.grantstutoring.com** to find out more about all of Grant's services. **Seminar Dates will be finalized no later than Sep. 25 for first term and Jan. 25 for second term.**

HOW TO USE THIS BOOK

I have broken the course up into lessons. Study each lesson until you can do all of my lecture problems from start to finish without any help. Then do the Practise Problems for that lesson. If you are able to solve all the Practise Problems I have given you, then you should have nothing to fear about your Midterm or Final Exam.

I have presented the course in what I consider to be the most logical order. Although my books are designed to follow the course syllabus, it is possible your prof will teach the course in a different order or omit a topic. It is also possible he/she will introduce a topic I do not cover. **Make sure you are attending your class regularly! Stay current with the material, and be aware of what topics are on your exam. Never forget, it is your prof that decides what will be on the exam, so pay attention.**

If you have any questions or difficulties while studying this book, or if you believe you have found a mistake, do not hesitate to contact me. My phone number and website are noted at the bottom of every page in this book. "Grant's Tutoring" is also in the phone book. **I welcome your input and questions.**

Wishing you much success,

Grant Skene

Owner of Grant's Tutoring

Formulas and Definitions to Memorize

Point-Slope form of a Line: $y - y_0 = m(x - x_0)$

Vertex of a Parabola: For all parabolas $y = ax^2 + bx + c$, the vertex is at $x = \frac{-b}{2a}$.

Compound Interest Formula: $A = P\left(1 + \frac{r}{m}\right)^{mt}$

Continuous Compounding Formula: $A = Pe^{rt}$

Exponential Growth or Decay Formula: $Q = Q_0 a^t$

The Definition of Continuity: $f(x)$ is continuous at $x=a$ if and only if $\lim_{x \to a} f(x) = f(a)$.

The Definition of Derivative: $f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$

The Power Rule: $(x^n)' = nx^{n-1}$

The Product Rule: $(f \cdot g)' = f'g + fg'$

The Quotient Rule: $\left(\frac{T}{B}\right)' = \frac{T'B - TB'}{B^2}$

The Chain Rule: $(f(u))' = f'(u) \cdot u'$

The Chain Rule Version of Power Rule: $(u^n)' = nu^{n-1} \cdot u'$

Lesson 2: Cost & Revenue, Demand & Supply

Lecture Problems:

(Each of the questions below will be discussed and solved in the lecture that follows.)

1. Find the equations of the following lines:

(a) The line with a slope of 5/3 and a y-intercept of –2.

(b) The line passing through the points (2, –3) and (6, 7).

(c) The line passing through the point (1, 1) and perpendicular to the line $2x - 7y = 5$.

(d) The line passing through the point (1, –1) and parallel to the line $y = 4x + 6$.

2. The cost C (in thousands of dollars) of a company that produces q widgets is given by $C = 12q + 40$.

(a) What is the cost of producing 50 widgets?

(b) How many widgets would cause a cost of $124,000?

3. The table below shows the number of inhabitants n (in thousands) in three cities and the amount of garbage produced each week g (in hundreds of metric tons).

n	20	25	40
g	17	35	89

(a) Does this data show a linear trend?

(b) Use this data to state g as a function of n.

4. In 1990, a company's sales were 20 million dollars. In 2000, they were 27 million dollars. Assuming the trend is linear, predict the sales in 2003.

5. A theatre has a fixed cost of \$3,000 per day and a variable cost of \$2 per customer. The admission fee is \$6 per customer.

(a) Find the cost and revenue functions. How many customers are needed to break even?

(b) Find the profit function and illustrate the break even point calculated in part (a) by sketching a graph of the profit function.

(c) What is the marginal cost, marginal revenue and marginal profit?

6. Let the demand curve for a certain product be $2p+q=100$ and the supply curve be $3p-q=50$ where p is the price in dollars and q is the quantity produced or sold. Find the equilibrium price. Illustrate this graphically.

7. The cost of producing widgets is $C(x)=x^2+8x+5$ where C is in dollars and x is measured in hundreds of widgets. Each widget sells for fourteen cents.

(a) How many widgets must be produced to break-even? Illustrate this point or points by graphing the cost and revenue functions.

(b) Find the profit in producing 200 widgets.

(c) Find the maximum profit and the production level that will attain it.

Lesson 3: Logs and Exponentials

Memorize the following formulas and facts:

e is a constant $(e = 2.718...)$ and "ln" is the log with base e:

$$\mathbf{\ln 1 = 0} \quad \text{and} \quad \mathbf{\ln \boldsymbol{e} = 1}$$

The 3 Log Laws:

$$\log_a(mn) = \log_a m + \log_a n \quad \rightarrow \quad \ln(mn) = \ln m + \ln n$$

$$\log_a\left(\frac{m}{n}\right) = \log_a m - \log_a n \quad \rightarrow \quad \ln\left(\frac{m}{n}\right) = \ln m - \ln n$$

$$\boxed{\log_a(m^p) = p \cdot \log_a m} \quad \rightarrow \quad \boxed{\ln(m^p) = p \cdot \ln m}$$

Compound Interest Formula: $A = P\left(1 + \frac{r}{m}\right)^{mt}$

Continuous Compounding Formula: $A = Pe^{rt}$

Exponential Growth or Decay Formula: $Q = Q_0 a^t$ **or** $Q = Q_0 e^{kt}$

Simplifying formulas with *e* and ln:

$$e^{\ln a} = a$$

$$e^{c \ln a} = e^{\ln a^c} = a^c$$

Lecture Problems:

(Each of the questions below will be discussed and solved in the lecture that follows.)

1. Solve for x in the equations below:

(a) $(3.4)^{2x} = 7$ **(b)** $6(5.2)^{x} = 24$ **(c)** $3x^{5} = 96$

(d) $x^{3/5} = 8$ **(e)** $\log_x 4 = \frac{1}{3}$ **(f)** $\log_5 x = 2$

2. You have $10,000 to invest at a 12% annual interest rate. How much money will you have at the end of 5 years if you are given:

(a) simple interest?

(b) interest compounded annually?

(c) interest compounded monthly?

(d) interest compounded continuously?

3. In terms of the natural log function, what is the tripling time of an investment deposited at 8% annual interest rate if it is compounded

(a) semi-annually **(b)** quarterly **(c)** continuously

4. How much must be invested at 6% compounded monthly if you need $25000 in 3 years?

5. If you need an investment to double in value in four years, what annual rate r, expressed as a percent, will you need if you are earning

(a) simple interest?

(b) interest compounded annually?

(c) interest compounded monthly?

(d) interest compounded continuously?

6. A bacteria colony is growing exponentially. Initially it had 500 cells. After 3 hours it had 11 thousand cells.

(a) How many cells will it have at the end of one day?

(b) How long will it take to reach one million cells?

7. A radioactive substance has a half-life of 1500 years.

(a) What percent of the substance will remain after 750 years?

(b) How long will it take to decay to 70% of its initial mass?

8. A radioactive substance has an initial mass of 300 grams and loses 50 grams of this mass over the next 100 days. What is its half-life if it is decaying

(a) linearly? **(b)** exponentially?

9. The amount of ozone in the atmosphere is decreasing exponentially at the rate of 0.9% per decade. How many years will it take until only half of the ozone remains?

10. A colony of bacteria is grown under ideal conditions in a laboratory so that the population increases exponentially with time. At the end of 3 hours, there are 10,000 bacteria. At the end of 5 hours there are 40,000 bacteria. How many bacteria were present initially?

Lesson 4: Limits

Lecture Problems:

(Each of the questions below will be discussed and solved in the lecture that follows.)

For each of **questions 1 to 14**, find the value of the limit, if it exists. If it does not exist, is it infinity, negative infinity, or neither? Justify your answers.

1. $\lim\limits_{x \to -1} \dfrac{x^2 - x - 2}{x + 1}$

2. $\lim\limits_{x \to -2} \dfrac{x + 2}{\sqrt{x + 4} - \sqrt{2}}$

3. $\lim\limits_{x \to 3} \dfrac{4 - \sqrt{x^2 + 7}}{4x^2 - 5x - 21}$

4. $\lim\limits_{x \to 2} \dfrac{|x - 2|}{x - 2}$

5. $\lim\limits_{x \to -1^+} \dfrac{x^2 - 3x - 4}{|5x^2 + 9x + 4|}$

6. $\lim\limits_{x \to 5} \dfrac{3 - x}{x^2 - 10x + 25}$

7. $\lim\limits_{x \to 2^-} \dfrac{x^2 - 9}{x^2 - x - 2}$

8. $\lim\limits_{x \to 6} \sqrt{x - 6}$

9. $\lim\limits_{x \to -2^-} \sqrt{x^2 - 7x - 18}$

10. $\lim\limits_{x \to \infty} \dfrac{2x^3 - x - 2}{1 + 3x^3}$

11. $\lim\limits_{x \to -\infty} \dfrac{4x^3 + 3x - 2}{3x^2 + 5x + 1}$

12. $\lim\limits_{x \to -\infty} \dfrac{5x - 21}{\sqrt{4x^2 - 3x + 7}}$

13. $\lim\limits_{x \to \infty} \dfrac{(2x - 3)^2 (x^2 + 6x + 5)}{x^4 + x^3 + 6}$

14. $\lim\limits_{x \to -\infty} \dfrac{(1 - 3x)\sqrt{4x^2 + 5}}{(5x + 4)^2}$

15. Which limits in questions 1 to 14 above indicate the existence of a Vertical or Horizontal Asymptote?

Lesson 5: Continuity

Memorize the Definition of Continuity:

$f(x)$ is continuous at $x=a$ if and only if $\lim_{x \to a} f(x) = f(a)$.

Lecture Problems:

(Each of the questions below will be discussed and solved in the lecture that follows.)

1. For the function shown below, determine for which x it is continuous. Justify your answer.

$$f(x) = \begin{cases} x-2 & \text{if} \quad x \le -2 \\ \dfrac{x^2-4}{x+2} & \text{if} \quad -2 < x < 1 \\ 4-x & \text{if} \quad 1 \le x \end{cases}$$

2. Is $g(x)$ below continuous at $x=1$? (Justify your answer.)

$$g(x) = \begin{cases} \dfrac{x-1}{\sqrt{x^2+3}-2} & x \ne 1 \\ 3 & x = 1 \end{cases}$$

3. For $f(x)$ below, find a and b which will make the function continuous everywhere.

$$f(x) = \begin{cases} x^2+4 & \text{if} \quad x \le 0 \\ ax+b & \text{if} \quad 0 < x < 3 \\ 6/x & \text{if} \quad x \ge 3 \end{cases}$$

4. Show that $f(x) = x^3 + 3x - 1$ has a zero between $x = 0$ and $x = 1$.

5. Show that $f(x) = x^3 - x^2 - 7x - 4$ has at least three zeros on $[-4, 4]$.

Lesson 6: The Definition of Derivative

Memorize the Definition of Derivative:

$$f'(x) = \lim_{h \to 0} \frac{f(x+h) - f(x)}{h}$$

Lecture Problems:

(Each of the questions below will be discussed and solved in the lecture that follows.)

1. For $f(x) = 2x^2 + 3x + 1$:

(a) Find the average rate of change of $f(x)$ for the interval $0 \le x \le 2$.

(b) Use the definition of derivative to find the instantaneous rate of change of $f(x)$.

(c) Find the equation of the tangent line to $y = f(x)$ at $x=1$ in $y = mx + b$ form.

2. For the functions below, find $f'(x)$ using only the definition of derivative.

(a) $f(x) = \dfrac{2x}{9 - x^2}$

(b) $f(x) = \dfrac{1}{\sqrt{x+1}}$

LESSON 7: DIFFERENTIATION RULES

All of the formulas on this page must be MEMORIZED

The Power Rule: $\left(x^n\right)' = nx^{n-1}$

The Product Rule: $(f \cdot g)' = f'\,g + f\,g'$

The Quotient Rule: $\left(\dfrac{T}{B}\right)' = \dfrac{T'\,B - T\,B'}{B^2}$

The Chain Rule: $\left(f(u)\right)' = f'(u) \cdot u'$

The Chain Rule Version of Power Rule: $\left(u^n\right)' = nu^{n-1} \cdot u'$

Derivatives of Trigonometric Functions (Optional):

$$(\sin u)' = \cos u \cdot u' \qquad (\cos u)' = -\sin u \cdot u'$$

Cost, Revenue and Profit:

Let $C(q)$, $R(q)$, $P(q)$ be, respectively, the cost, revenue and profit of producing and selling q units then:

Marginal Cost, $C'(q)$, is the expected cost of producing the next unit.
Marginal Revenue, $R'(q)$, is the expected revenue from selling the next unit.
Marginal Profit, $P'(q)$, is the expected profit from producing and selling the next unit.

Average Cost is $\dfrac{C(q)}{q}$**, Average Revenue is** $\dfrac{R(q)}{q}$ **and Average Profit is** $\dfrac{P(q)}{q}$**.**

Lecture Problems:

(Each of the questions below will be discussed and solved in the lecture that follows.)

1. Find the indicated derivatives for the following. You need not simplify your answer.

(a) $y = \sqrt[3]{x^2} + \frac{2}{x^3} + \frac{4x}{5}$, find $\frac{dy}{dx}$

(b) $y = \sqrt{2} - 5x^2 + \frac{12}{5x^7}$, find y'

(c) $y = \frac{x + 9x^5 + 1}{3x + 2}$, find y'

(d) $y = \left(x^2 + 1\right)^{10}$, find $\frac{dy}{dx}$

(e) $y = \left(4 - 6x^2\right)^8$, find y'

(f) $y = 2x^7\sqrt{x^2 + 1}$, find y'

(g) $y = e^{2x+1}$, find y'

(h) $f(t) = e^{3t^2} + 4t^3$, find $f'(1)$

(i) $f(x) = \sqrt[3]{x^4 + \left(x + x^3\right)^{-2}}$, find $f'(x)$

(j) $y = (x+1)\left(x^2+1\right)^3\left(x^3-1\right)^4$, find y'

(k) $y = \frac{x^3\sqrt[5]{x+2}}{x^3 + 2x + 1}$, find y'

(l) $y = \left(x^2 + \pi^2\right)\left(\sqrt[5]{\left(x^2 + x + 3\right)^4}\right)$, find y'

2. Find an equation of the tangent line to the curve $y = \left(x^3 + x^2 + 2\right)^5$ when $x = -2$.

3. Find the equation of the normal line to the curve whose equation is $y = 2 + \sqrt{x^2 + 3}$ at the point on the curve with x-coordinate 1.

4. Given $f(x) = (2x + 5)^3$, compute $f''(-2)$.

5. The sales of a company are related to its expenditures on research by the formula $S(x) = 1000 + 50\sqrt{x} + 10x$, where $S(x)$ gives sales in millions when x thousand dollars is spent on research. Find and interpret $\frac{dS}{dx}$ if the following amounts are spent on research:

(a) \$9,000 **(b)** \$16,000 **(c)** \$25,000

(d) What happens to sales as the amount on research increases?

6. An analyst has found that a company's costs and revenues in dollars for one product are given by $C(x) = 2x$ and $R(x) = 6x - \frac{x^2}{1000}$, respectively, where x is the number of items produced.

(a) Find the marginal cost function.

(b) Find the marginal revenue function.

(c) Find the marginal profit function.

(d) Find and interpret the value of x that makes the marginal profit equal to 0.

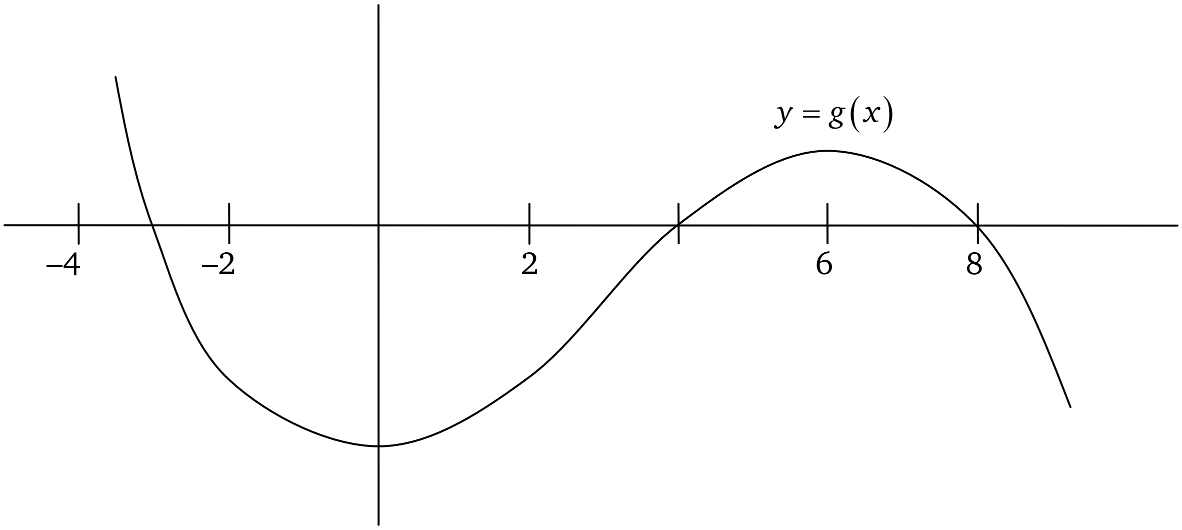
7. If the price in dollars of a stereo system is given by $p(q) = \frac{1000}{q^2} + 1000$, where q represents the demand for the product, find the marginal revenue when the demand is 10.

8. A point moves along the x-axis such that its position at any time t is given by the function $x = 5t^2 - 3t^3$. Find the position, velocity and acceleration of the point when $t=2$.

9. We are given $y = f(x^2 + x)$, where f is an unknown differentiable function.

We know $f(2) = -5$ and $f'(2) = 6$. Compute $\left.\frac{dy}{dx}\right|_{x=1}$.

10. For the graph of $y = g(x)$ as shown determine whether the following quantities are positive, negative or zero.



(a) $g(-3)$

(b) $g'(-3)$

(c) $g(0)$

(d) $g'(0)$

(e) $g(2)$

(f) $g'(2)$

(g) $\dfrac{g(8)-g(2)}{8-2}$

(h) $\dfrac{g(8)-g(6)}{2}$

(i) $\lim\limits_{h\to 0}\dfrac{g(4+h)-g(4)}{h}$

(j) $\lim\limits_{h\to 0}\dfrac{g(6+h)-g(6)}{h}$