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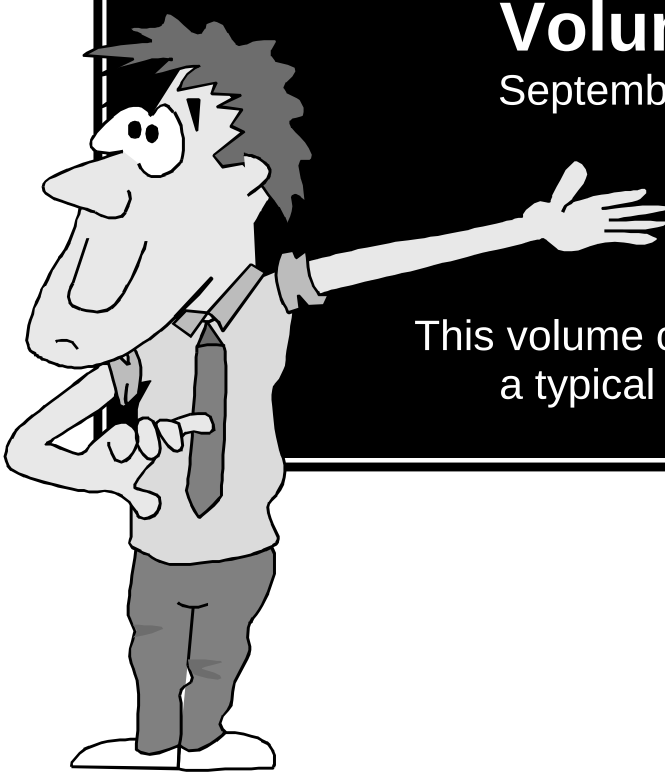

BASIC STATISTICS 2

Volume 1 of 2

September 2011 edition

This volume covers the topics on
a typical midterm exam.

Learn What You Need to Know
Know What You Need to Learn

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HOW TO USE THIS BOOK

I have broken the course up into lessons. Do note that the numbering of my lessons do not necessarily correspond to the numbering of the units in your course outline. Study each lesson until you can do all of my lecture problems from start to finish without any help. Then do the Practise Problems for that lesson. If you are able to solve all the Practise Problems I have given you, then you should have nothing to fear about your exams.

I also recommend you purchase the *Multiple-Choice Problems Set for Basic Statistical Analysis II (Stat 2000)* by Dr. Smiley Cheng available at The Book Store. The appendices of my book include complete step-by-step solutions for all the problems and exams in Cheng's book. Be sure to read the "Homework" section at the end of each lesson for important guidance on how to proceed in your studying.

You also need a good, but not expensive, scientific calculator. Any of the makes and models of calculators I discuss in Appendix A are adequate for this course. I give you more advice about calculators at the start of Lesson 1. **Appendix A in this book shows you how to use all major models of calculators.**

I have presented the course in what I consider to be the most logical order. Although my books are designed to follow the course syllabus, it is possible your prof will teach the course in a different order or omit a topic. It is also possible he/she will introduce a topic I do not cover. **Make sure you are attending your class regularly! Stay current with the material, and be aware of what topics are on your exam. Never forget, it is your prof that decides what will be on the exam, so pay attention.**

If you have any questions or difficulties while studying this book, or if you believe you have found a mistake, do not hesitate to contact me. My phone number and website are noted at the bottom of every page in this book. "Grant's Tutoring" is also in the phone book. **I welcome your input and questions.**

Wishing you much success,

Grant Skene

Owner of Grant's Tutoring and author of this book

FORMULA SHEET

A formula sheet is included in your exams. Check your course syllabus and compare it to the formula sheet I use below in case the formula sheet in your course has changed.

1. $SE\left(\bar{x}_1 - \bar{x}_2\right) = \sqrt{\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}}$ with $\text{df} = \dfrac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2}$

2. $SE\left(\bar{x}_1 - \bar{x}_2\right) = s_p\sqrt{\frac{1}{n_1} + \frac{1}{n_2}}$ with $\text{df} = n_1 + n_2 - 2$ if $\sigma_1^2 = \sigma_2^2$

where $s_p^2 = \dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2}{n_1 + n_2 - 2}$

3. $SSG = \sum_{i=1}^{k} n_i \left(\bar{X}_i - \bar{\bar{X}}\right)^2$

4. Poisson Distribution: $P(X = k) = \dfrac{e^{-\lambda}\lambda^k}{k!}$ $\quad k = 0, 1, 2, \ldots$

5. $t = \dfrac{r\sqrt{n-2}}{\sqrt{1 - r^2}}$

6. $SE_{b_1} = \dfrac{s_e}{\sqrt{\sum (x_i - \bar{x})^2}}, \quad s_e = \sqrt{MSE}$

7. $SE_{b_0} = s_e\sqrt{\frac{1}{n} + \frac{\bar{x}^2}{\sum (x_i - \bar{x})^2}}$

8. $SE_{\hat{\mu}} = s_e\sqrt{\frac{1}{n} + \frac{(x^* - \bar{x})^2}{\sum (x_i - \bar{x})^2}}$

9. $SE_{\hat{y}} = s_e\sqrt{1 + \frac{1}{n} + \frac{(x^* - \bar{x})^2}{\sum (x_i - \bar{x})^2}}$

10. $SE\left(\hat{p}_1 - \hat{p}_2\right) = \sqrt{\hat{p}(1 - \hat{p})\left(\frac{1}{n_1} + \frac{1}{n_2}\right)}$ if $p_1 = p_2$ where $\hat{p} = \dfrac{x_1 + x_2}{n_1 + n_2}$

11. $SE\left(\hat{p}_1 - \hat{p}_2\right) = \sqrt{\frac{\hat{p}_1(1 - \hat{p}_1)}{n_1} + \frac{\hat{p}_2(1 - \hat{p}_2)}{n_2}}$ if $p_1 \neq p_2$

STEPS FOR TESTING A HYPOTHESIS

Step 1. State the null and alternative hypotheses (H_0 and H_a), and so determine if the test is 2-tailed, upper-tailed, or lower-tailed.

Step 2. Use the given α (always use α = 5% if none is given) to get the **critical value** (z^*, t^*, F^*, etc. depending on the hypothesis you are testing) from the appropriate table and state the **rejection region**.

Step 3. Compute the **test statistic** (z, t, F, etc. depending on the hypothesis you are testing) using the appropriate formula, and see if it lies in the rejection region.

Step 4. (Only if specifically asked to do so.) Compute the ***P*-value**.

Draw a density curve (z-bell curve, t-bell curve, F right-skewed curve, etc. depending on the test statistic you have computed), mark the test statistic (found in Step 3), and shade the area as instructed by H_a. That area is the P-value.

Remember, a P-value is very handy to know if you are asked to make decisions for more than one value of α.

Reject H_0 if P-value < α.

Step 5. State your conclusion.

Either: Reject H_0. There is statistically significant evidence that the alternative hypothesis is correct. (Replace the underlined part with appropriate wording from the problem that says H_a is correct.)

Or: Do not reject H_0. There is no statistically significant evidence that the alternative hypothesis is correct. (Replace the underlined part with appropriate wording from the problem that says we are not convinced that H_a is correct.)

SUMMARY OF KEY FORMULAS IN THIS COURSE

Lesson 1. The mean and standard deviation of $\bar{x}$ are $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$.

The standard error of $\bar{x} = SE_{\bar{x}} = \frac{s}{\sqrt{n}}$.

Central Limit Theorem: If n is large, $\bar{x}$ is approximately normal.

Confidence Intervals for μ: $\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}}$ or $\bar{x} \pm t^* \frac{s}{\sqrt{n}}$

Sample size determination: $n = \left(\frac{z^* \sigma}{m}\right)^2$

Lesson 2. Test statistics for H_0: $\mu = \mu_0$ are $z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$ or $t = \frac{\bar{x} - \mu_0}{s / \sqrt{n}}$.

Lesson 3. Standardizing formula for $\bar{x}$ bell curves: $z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}}$.

To compute $\bar{x}^*$ for $\bar{x}$ decision rules: $\bar{x}^* = z^* \frac{\sigma}{\sqrt{n}} + \mu_0$.

Lesson 4. Properties for means of two random variables:

$$\mu_{X+Y} = \mu_X + \mu_Y$$

$$\mu_{X-Y} = \mu_X - \mu_Y$$

Properties for variance of two independent random variables:

$$\sigma^2_{X+Y} = \sigma^2_X + \sigma^2_Y$$

$$\sigma^2_{X-Y} = \sigma^2_X + \sigma^2_Y$$

Properties for variance of two dependent random variables with correlation ρ:

$$\sigma^2_{X+Y} = \sigma^2_X + \sigma^2_Y + 2\rho\sigma_X\sigma_Y$$

$$\sigma^2_{X-Y} = \sigma^2_X + \sigma^2_Y - 2\rho\sigma_X\sigma_Y$$

Confidence interval for $\mu_1 - \mu_2$ is $(\bar{x}_1 - \bar{x}_2) \pm t^* SE(\bar{x}_1 - \bar{x}_2)$.

To test H_0: $\mu_1 = \mu_2$, the test statistic is $t = \frac{\bar{x}_1 - \bar{x}_2}{SE(\bar{x}_1 - \bar{x}_2)}$.

The formulas for the degrees of freedom and $SE(\bar{x}_1 - \bar{x}_2)$ are included on the Formula Sheet given on your exams (page 1 of this book).

SUMMARY OF KEY FORMULAS IN THIS COURSE (CONTINUED)

Lesson 5. $DFG = I - 1$ and $DFE = N - I$

$$\bar{\bar{x}} = \frac{\sum n_i \bar{x}_i}{N} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + n_3\bar{x}_3 + \ldots + n_I\bar{x}_I}{N}$$

$MSG = \dfrac{SSG}{DFG}$ (The formula for SSG is included on the Formula Sheet given on your exams (page 1 of this book).)

$$MSE = \frac{SSE}{DFE} = \frac{\sum (n_i - 1)s_i^2}{N - I} = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2 + \ldots + (n_I - 1)s_I^2}{N - I}$$

$s_p^2 = MSE$. (The Formula Sheet given on your exams (page 1 of this book) gives you the two-sample version of the s_p^2 formula; this formula can be generalized for three, four or more samples.)

The formula for the F test statistic is $F = \dfrac{MSG}{MSE}$ with df = DFG, DFE.

The coefficient of determination = $R^2 = \dfrac{SSG}{SST}$.

Lesson 6. If X is a discrete random variable:

$$\mu = \sum x\, p(x) \text{ and } \sigma^2 = \sum (x - \mu)^2 p(x) \text{ or } \sigma^2 = \left(\sum x^2 p(x)\right) - \mu^2$$

For a **binomial distribution** $P(X = k) = \binom{n}{k} p^k (1 - p)^{n-k}$. (This formula does not really have to be memorized as it is included at the top of Table C which you will be given on exams.)

For a binomial distribution with parameters n and p:

$$\boldsymbol{\mu_X = np} \text{ and } \boldsymbol{\sigma_X = \sqrt{np(1-p)}}.$$

If we are using the normal approximation to the binomial distribution, we can standardize the random variable X into z scores using $z = \dfrac{x - \mu}{\sigma}$.

For a **Poisson distribution** $P(X = k) = \dfrac{e^{-\lambda}\lambda^k}{k!}$. (This formula does not really have to be memorized since it is included on your formula sheet (#4).

The Poisson distribution has only one parameter, λ:

$$\boldsymbol{\mu_X = \lambda} \text{ and } \boldsymbol{\sigma_X^2 = \lambda}, \text{ so } \boldsymbol{\sigma_X = \sqrt{\lambda}}.$$

SUMMARY OF KEY CONCEPTS IN LESSON 1

- Populations have **parameters**; samples have **statistics**.
- The **Law of Large Numbers** states:
  - ***As n gets larger, any statistic will come closer and closer to the value of the parameter it is estimating. The statistic will have less and less variability.***
  - For example, as n gets larger, $\bar{x}$ will come closer and closer to μ.
- The mean of $\bar{x}$ = $\mu_{\bar{x}} = \mu$ and the standard deviation of $\bar{x}$ = $\sigma_{\bar{x}} = \frac{\sigma}{\sqrt{n}}$.
- If the population is normal (we have an X-bell curve), then the sample mean is also normally distributed (we have an $\bar{x}$-bell curve).
- The **Central Limit Theorem** states:
  - ***As n gets larger, the distribution of the sample mean becomes closer and closer to the normal distribution.***
  - Even if the population is not normal, $\bar{x}$ will have an approximately normal distribution as long as n is large. Usually, $n \geq 15$ is large enough for the Central Limit Theorem to apply.
- A **confidence interval for the population mean,** μ**,** can be computed using either:

$$\bar{x} \pm z^* \frac{\sigma}{\sqrt{n}} \qquad \text{or} \qquad \bar{x} \pm t^* \frac{s}{\sqrt{n}}$$

- **If σ is given, we can use z; if σ is not given we must use t.**
- The t distribution is a bell-shaped density curve centred at 0 but of a varying spread. The spread depends on the degrees of freedom. The larger the degrees of freedom get, the narrower the spread of the t curve. Ultimately, the t curve will look exactly like the standard normal curve z.
- When using t our **df = n – 1**. This is because t is using the sample standard deviation s, since σ is unknown, to compute the standard error of the sample mean.
  - The standard error of the sample mean is $SE_{\bar{x}} = \frac{s}{\sqrt{n}}$.

- When listing givens: if n is given in a sentence, you know you are being given statistics (sample info) such as the *sample* mean $\bar{x}$ or the *sample* standard deviation s. If n is not given in the sentence, you are being given parameters (population info) such as the *population* mean μ or the *population* standard deviation σ.
- We say that **inferences about the mean are robust** because we can generally trust them to give us reliable confidence intervals for μ and trustworthy conclusions from our hypotheses for μ (next lesson) even if our population is not normal. All we need is a random sample and a large enough sample size.
- Inferences about the mean are only reliable if the sample mean $\bar{x}$ is normally distributed. We know $\bar{x}$ is normal if the population is normal. If the population is not normal, $\bar{x}$ will still be approximately normal as long as n is large (by Central Limit Theorem). In general, as long as $\boldsymbol{n \geq 15}$, we can assume $\bar{x}$ is approximately normal. If we believe the population has outliers or is strongly skewed, we want $\boldsymbol{n \geq 40}$ to be able to safely assume $\bar{x}$ is approximately normal.
- The *higher* our level of confidence, the *wider* our confidence interval becomes. Put another way, **as we increase our confidence level, our margin of error increases also**.
- The *larger* our sample size, the *narrower* our confidence interval becomes. Put another way, **as we increase our sample size, our margin of error decreases**.
- The larger the standard deviation we are using, the larger the margin of error of our confidence interval will be.
- Everything that comes after the "±" in a confidence interval formula is computing the margin of error m. There are three parts to the margin of error in a confidence interval for the mean:
  - The **critical value (z^* or t^*)** which is dictated by the **level of confidence** and is read off Table D (the t distribution table).
  - The **standard deviation (σ or s)**.
  - The **sample size (n)**.
- When we construct, for example, a 95% confidence interval for the mean, we are saying we are 95% confident the true value of the mean, μ, is somewhere in that interval. We never know for sure if our interval has indeed caught μ, but we do know that, **if we repeatedly took samples of the same size, and repeatedly used those samples to construct 95% confidence intervals for μ then, in the long run, 95% of the intervals we construct will contain the true mean, μ.** This is because we are assuming our sample mean, $\bar{x}$, is in the middle 95% of the curve. Since 95% of all the possible $\bar{x}$ values are in this region, 95% of the confidence intervals we construct will contain the population mean μ.

- **If we are using z^* and <u>if the level of confidence we are using is not given on Table D</u>, we have to find z^* by reading Table A backwards. The level of confidence is the percentage shaded in the *middle* of the z-bell curve, leaving us with two equally sized tails. Establish the area in the left tail (knowing the total area is 100%), then look for the closest value you can find to that left tail area in the body of Table A. That will give us the negative z-score. Discard the negative sign and you have z^*.** This would never happen if you are using t^*. You can rest-assured the level of confidence you are given will always be on Table D when you are using t^*.
- If the level of confidence you desire is not on Table D, you can get a rough estimate for z^* by averaging the two z^* values on either side of that level of confidence. This is a quick way of getting an answer that, although inaccurate, will be close enough for you to ascertain the correct choice in a multiple choice question.
- To determine the sample size required to obtain a specific margin of error m for a confidence interval for μ, we use the sample size formula:

$$n = \left(\frac{z^* \sigma}{m}\right)^2$$

- Always use the **Paint-Can Principle** when computing sample size. Which is to say, you must always round UP↑ to the next whole number. Even if you have computed $n = 58.1$, you must round that up to $n = 59$ because 58 units are not enough, you need 58.1 units, so you will have to select 59 units.

LECTURE PROBLEMS FOR LESSON 1

For your convenience, here are the 10 questions I used as examples in this lesson. Do not make any marks or notes on these questions below. Especially, do not circle the correct choice in the multiple choice questions. You want to keep these questions untouched, so that you can look back at them without any hints. Instead, make any necessary notes, highlights, etc. in the lecture part above.

1. We select a random sample of 25 adult males and find their mean weight to be 67.53 kg. Assume the population is normal with a standard deviation of 10 kg.

(a) A 95% confidence interval for the mean weight of adult males is

(A) (63.61, 71.45) **(B)** (62.73, 72.23) **(C)** (63.40, 71.66)
(D) (61.61, 73.45) **(E)** (65.00, 70.06)

(See the solution on page 31.)

(b) Interpret this confidence interval including an explanation of what 95% confidence really means so that a layman might understand.

(See the solution on page 32.)

2. An SRS of 25 adult Canadians finds they have an average annual income of 30 thousand dollars with a standard deviation of 11 thousand dollars. A 95% confidence interval for the true mean income is

(A) (29.120, 31.980) **(B)** (25.688, 34.312) **(C)** (25.459, 34.541)
(D) (25.468, 34.532) **(E)** (26.236, 33.764)

(See the solution on page 33.)

3. It is known a certain machine that fills "5-pound" sacks of sugar actually fills the sacks with an amount that varies from sack to sack with variance 0.0016 pounds squared. Suppose a sample of 30 sacks has an average weight of 5.08 pounds. A 98% confidence interval estimate for the mean of the population of all sacks filled by the process in its current state is:

(A) $5.08 - 1.960(.0073) \le \mu \le 5.08 + 1.960(.0073)$

(B) $5.08 - 2.462(.0040) \le \mu \le 5.08 + 2.462(.0040)$

(C) $5.08 - 2.326(.0040) \le \mu \le 5.08 + 2.326(.0040)$

(D) $5.08 - 2.326(.0073) \le \mu \le 5.08 + 2.326(.0073)$

(E) $5.08 - 2.462(.0073) \le \mu \le 5.08 + 2.462(.0073)$

(See the solution on page 34.)

4. Investigating toxins produced by molds that infect corn crops, a biochemist prepares extracts of the mold culture and measures the amount of the toxic substance. From six preparations, the following observations on toxic substances per gram of solution are obtained: 1.2, 0.8, 0.6, 1.1, 1.2, 1.1. The endpoints of a 95% confidence interval for the mean amount of toxic substances are (assuming a normal population):

(A) $1.0 \pm 2.571(0.10)$ **(B)** $1.0 \pm 1.960(0.10)$ **(C)** $.95 \pm 2.571(0.10)$
(D) $.95 \pm 1.960(0.10)$ **(E)** $1.0 \pm 2.447(0.10)$

(See the solution on page 35.)

5. A manufacturer of bran flakes claims a 30 gram serving of its cereal has a mean of 4.4 grams of dietary fibre with standard deviation of 0.25 grams. Five random 30 gram servings have the following amounts of dietary fibre: 4.5, 4.4, 4.3, 4.4, 3.9. Assuming the population is normally distributed with given standard deviation, a 90% confidence interval for the true mean amount of dietary fibre is

(A) $4.3 \pm 1.645(.105)$ **(B)** $4.3 \pm 2.132(.105)$ **(C)** $4.3 \pm 2.132(.112)$
(D) $4.3 \pm 1.645(.112)$ **(E)** $4.4 \pm 1.645(.112)$

(See the solution on page 36.)

6. We are investigating the lifetimes of a brand of batteries and we know the standard deviation is 8 hours. How large a sample size is necessary to estimate the average lifetime within 2.5 hours with 95% confidence?

(A) 39 **(B)** 40 **(C)** 16 **(D)** 256 **(E)** 1000

(See the solution on page 38.)

7. A company that manufactures O-rings for the space shuttle knows that when the manufacturing process is stable the critical dimension has a standard deviation equal to 0.065 millimetre and that the distribution of the measurements look very similar to the normal distribution. The sample size that would be required to estimate the process mean, μ, so that a 99% confidence interval would have width 0.036 mm is

(A) 4 **(B)** 14 **(C)** 50 **(D)** 51 **(E)** 87

(See the solution on page 39.)

8. A 95% confidence interval for a normal population with known standard deviation produces a margin of error of .004 based on a sample size of 100. If we wish to halve the margin of error (maintaining the same level of confidence), how large a sample must we take?

(A) 50 **(B)** 100 **(C)** 200 **(D)** 400

(E) cannot be determined without the variance

(See the solution on page 40.)

9. A confidence interval for the mean of a population with known variance is to be constructed. Which of the following statements is FALSE?

(A) A confidence interval using a sample size of 100 will be wider than a confidence interval using a sample size of 50.

(B) A 99% confidence interval will be wider than a 90% confidence interval.

(C) The larger the variance, the wider the confidence interval will be.

(D) A 90% confidence interval will be narrower than a 95% confidence interval.

(E) A confidence interval using a sample size of 100 will be narrower than a confidence interval using a sample size of 50.

(See the solution on page 41.)

10. A random sample of 15 car batteries maintained their charge for an average of 36.7 hours in laboratory tests. Assuming the population is normal with a standard deviation of 7 hours, a 92% confidence interval for the average length of time car batteries will maintain their charge is

(A) (33.16, 40.24) **(B)** (33.54, 39.86) **(C)** (33.73, 39.67)

(D) (32.04, 41.36) **(E)** impossible to determine with the tables we have

(See the solution on page 43.)

SUMMARY OF KEY CONCEPTS IN LESSON 2

- All hypothesis tests for the mean, μ, are reliable provided we are dealing with representative samples and that the sample mean, $\bar{x}$, is normally distributed. We want random samples and properly designed experiments to be assured of a representative sample. If the population is normal, we certainly know $\bar{x}$ is normally distributed. If the population is not normal, $\bar{x}$ will be approximately normal if n is large (by Central Limit Theorem).
- Memorize the **five steps to test a hypothesis** (listed on page 72).
- The **critical value** is either z^* or t^* and is read off of Table D.
- The **test statistic** is computed from the appropriate formula.

$$z = \frac{\bar{x} - \mu_0}{\sigma/\sqrt{n}} \qquad \text{or} \qquad t = \frac{\bar{x} - \mu_0}{s/\sqrt{n}}$$

- The ***P*-value** tells us the probability of getting our test statistic or something even more extreme, in the appropriate direction, assuming the null hypothesis is correct.
- The smaller the *P*-value is, the stronger the evidence to reject the null hypothesis.
- If you are ever given more than one level of significance α in a problem, compute the *P*-value so that you can make your decision more easily.
- **Reject H_0 if the *P*-value $< \alpha$.**
- **If you are given a confidence interval, you can use it to test a two-tailed hypothesis.** Confidence intervals are of no use for one-tailed hypotheses.
  - The level of confidence dictates the level of significance you must use.
  - $\alpha = 1 - C$
  - If the hypothesized value μ_0 is inside the confidence interval for μ, we cannot reject H_0 since it is possible that H_0 is correct. If μ_0 is not inside the interval, we can reject H_0 since we are confident H_0 is wrong.

LECTURE PROBLEMS FOR LESSON 2

For your convenience, here are the 18 questions I used as examples in this lesson. Do not make any marks or notes on these questions below. Especially, do not circle the correct choice in the multiple choice questions. You want to keep these questions untouched, so that you can look back at them without any hints. Instead, make any necessary notes, highlights, etc. in the lecture part above.

1. State the null and alternative hypotheses for the following problems. Also, state whether the test is lower-tailed, upper-tailed, or two-tailed.

(a) A real estate broker trying to sell a business claims the average daily receipts are at least \$3200, we want to test this claim.

(b) An engineer declares it will take an average of 150 man-hours to manufacture an airplane part, we want to see if he is right.

(c) A pizza restaurant claims it will deliver your pizza in an average of 30 minutes or less, you want to know if this is true.

(d) A tutor claims his students will score more than 70, on average, on their tests.

(See the solution on page 59.)

2. For the situations below, state the rejection region. Assume the populations are normally distributed.

(a) H_0: $\mu = 25$ vs. H_a: $\mu > 25$; $\sigma = 6$; $n = 30$; $\alpha = 5\%$.

(b) H_0: $\mu = 35$ vs. H_a: $\mu > 35$; σ is unknown; $n = 30$; $\alpha = 5\%$.

(c) H_0: $\mu = 25$ vs. H_a: $\mu < 25$; $\sigma = 16$; $n = 20$; $\alpha = 1\%$.

(d) H_0: $\mu = 25$ vs. H_a: $\mu \neq 25$; $\sigma = 6$; $n = 10$; $\alpha = 5\%$.

(e) H_0: $\mu = 25$ vs. H_a: $\mu \neq 25$; σ is unknown; $n = 18$; $\alpha = 1\%$.

(f) H_0: $\mu = 35$ vs. H_a: $\mu < 35$; σ is unknown; $n = 10$; $\alpha = 5\%$.

(See the solution on page 64.)

3. Return to question 1 above. Let us assume that, in each of parts (a) through (d), we decided to reject H_0. Give a carefully worded conclusion relevant to the problem. In addition, state the conclusion if we did not reject H_0 in each case.

(See the solution on page 69.)

4. The scores of students who wrote a provincial exam in mathematics are normally distributed with mean 75 and variance 36. A teacher believes his group of students is above average. The 25 students in his class scored an average of 79. Is the teacher's belief justified? Include the hypotheses, critical value, and test statistic in your answer.

(See the solution on page 74.)

5. A manufacturer of small electric motors asserts they will draw 0.8 amperes, on average, under normal load conditions. A sample of 16 of the motors was tested and it was found the mean current was 0.96 amperes with a standard deviation of 0.32 amperes. Assume the sample is normally distributed. At the 1% level of significance, are we justified in rejecting the manufacturer's assertion? Include the hypotheses, critical value, test statistic, and a carefully worded conclusion relevant to this problem in your answer.

(See the solution on page 79.)

6. Compute the P-value for the hypothesis tests you conducted in questions 4 and 5 above. Would the results be significant if you used a 10% level of significance for both? What if you used a 1% level of significance for both? Interpret the P-values for questions 3 and 4.

(See the solution on page 90.)

7. The P-value of a test of a null hypothesis is

(A) the probability, assuming the null hypothesis is true, that the test statistic will take a value at least as extreme (in the appropriate direction) as that actually observed.

(B) the probability, assuming the null hypothesis is false, that the test statistic will take a value at least as extreme (in the appropriate direction) as that actually observed.

(C) the probability that the null hypothesis is true.

(D) the probability that the null hypothesis is false.

(E) either 0.01, or 0.05, or 0.10.

(See the solution on page 91.)

8. In testing the hypothesis H_0: $\mu = 10$ versus H_a: $\mu \neq 10$ at $\alpha = .05$, we know the standard deviation is 6. A sample of size 36 has a mean of 7.87. The P-value for the test and our decision is:

(A) .0166; reject H_0. **(B)** .0166; fail to reject H_0. **(C)** .0332, reject H_0.

(D) .0332, fail to reject H_0. **(E)** none of the above.

(See the solution on page 92.)

9. A real estate broker trying to sell a luxury apartment block claims that the average size of the apartments is at least 3200 square feet. You are sceptical and measure 16 random apartments, finding the size to be an average of 2984 square feet with a standard deviation of 420 square feet. The P-value for the appropriate hypothesis test would be:

(A) between .025 and .01 **(B)** between .05 and .025

(C) between .10 and .05 **(D)** .0394 **(E)** .0197

(See the solution on page 93.)

10. A manufacturer of bran flakes claims a 30 gram serving of its cereal has a mean of 4.4 grams of dietary fibre with standard deviation of 0.25 grams. Five random 30 gram servings have the following amounts of dietary fibre: 4.2, 4.1, 4.3, 4.4, 3.9. Assuming the population is normally distributed with given standard deviation, does this sample give convincing evidence the mean amount of dietary fibre is less than claimed? Use a .05 level of significance.

(a) State the hypotheses.

(A) H_0: $\mu = 4.4$ vs. H_a: $\mu \neq 4.4$ **(B)** H_0: $\mu = 4.3$ vs. H_a: $\mu < 4.3$

(C) H_0: $\bar{x} = 4.18$ vs. H_a: $\bar{x} < 4.18$ **(D)** H_0: $\mu = 4.4$ vs. H_a: $\mu > 4.4$

(E) H_0: $\mu = 4.4$ vs. H_a: $\mu < 4.4$

(See the solution on page 94.)

(b) We would reject the null hypothesis if

(A) $t < -2.132$ **(B)** $t > 2.132$ **(C)** $z < -1.645$

(D) $|z| > 1.96$ **(E)** $z > 1.645$

(See the solution on page 95.)

(c) The value of the test statistic is

(A) –1.97 **(B)** –1.645 **(C)** –2.56 **(D)** –1.95 **(E)** 1.645

(See the solution on page 95.)

(d) The *P*-value of the test is

(A) 0.0512 **(B)** 0.0488 **(C)** 0.0244

(D) 0.0256 **(E)** between 0.05 and 0.25

(See the solution on page 95.)

(e) The conclusion is

(A) Don't reject H_0. There is convincing evidence the mean amount of fibre is 4.4 grams.

(B) Reject H_0. There is convincing evidence the mean amount of fibre is not 4.4 grams.

(C) Don't reject H_0. There is no convincing evidence the mean amount of fibre is less than 4.4 grams.

(D) Don't reject H_0. There is no convincing evidence the mean amount of fibre is not 4.4 grams.

(E) Reject H_0. There is convincing evidence the mean amount of fibre is less than 4.4 grams.

(See the solution on page 96.)

11. The average score for students who complete a university statistics course is 55. A random sample of 20 students who used a certain tutor's services to help in this course scored an average of 61 with a standard deviation of 10. Assuming the distribution is normal, perform a test of significance using the *P*-value approach to determine if the tutor has a positive effect. Be sure to include the hypotheses, test statistic, *P*-value, and a properly worded conclusion in your answer.

(See the solution on page 98.)

12. The average wait time for an MRI in the year 2000 was 240 days with a standard deviation of 60 days. A random sample of 40 patients who received an MRI last year found that they waited an average of 220 days. Can the government claim they have reduced waiting times? Use the *P*-value approach and assume the standard deviation is unchanged. Be sure to include the hypotheses, test statistic, *P*-value, and a properly worded conclusion in your answer.

(See the solution on page 100.)

13. If (2.5, 4.7) is an observed 99% confidence interval for the parameter μ.

(a) Which of these statements is true?

(A) The probability a sample lies between 2.5 and 4.7 is 0.99.
(B) The probability the sample mean lies between 2.5 and 4.7 is 0.99.
(C) 99% of the samples lie between 2.5 and 4.7.
(D) 99% of the true means μ lie between 2.5 and 4.7
(E) If many random samples (each of the same size) were taken, and a 99% C.I. calculated using each sample, then (in the long run) 99% of them would contain the unknown μ.

(See the solution on page 102.)

(b) A possible 95% confidence interval constructed from the same information is

(A) (2.3, 4.7) **(B)** (2.7, 4.5) **(C)** (2.7, 4.7) **(D)** (2.3, 4.9) **(E)** (2.5, 4.9)

(See the solution on page 104.)

(c) The sample mean used to construct this confidence interval is

(A) 1.96 **(B)** 2.5 **(C)** 4.7 **(D)** 3.6 **(E)** impossible to determine.

(See the solution on page 105.)

(d) We want to test H_0: $\mu = 2.6$ vs. H_a: $\mu \neq 2.6$. With this information we definitely

(A) reject H_0 at the 1% level of significance.
(B) fail to reject H_0 at the 5% level of significance.
(C) fail to reject H_0 at the 2.5% level of significance.
(D) fail to reject H_0 at the 1% level of significance.
(E) are unable to make a decision.

(See the solution on page 107.)

14. In a test of hypotheses, how should the level α be chosen if we want very strong evidence against the null hypothesis H_0 before rejecting it?

(A) very small **(B)** very large **(C)** smaller than the P-value
(D) larger than the P-value **(E)** none of the above

(See the solution on page 108.)

15. Suppose we have rejected the null hypothesis H_0: $\mu = \mu_0$, at a significance level $\alpha = .01$, where μ is the mean of a normal distribution with known variance. Now, using the same sample information, we want to test the same null hypothesis, but at significance level $\alpha = .05$.

(A) We will reject H_0.
(B) We will fail to reject H_0.
(C) We cannot make any decision until we obtain the value from the table.
(D) We cannot make any decision until we calculate the value of the test statistic.
(E) Both (C) and (D).

(See the solution on page 109.)

16. It is imperative an important part of a jet engine measure an average of 224 mm with a standard deviation of .00400 mm. Assuming the measures of these parts are normally distributed with the required standard deviation, the quality control inspector examines a random samples of 16 using *JMP*™. Here are her results:

Jet Engine Part

Moments		Test Mean=value	
Mean	224.0019	Hypothesized Value	224
Std Dev	0.0618	Actual Estimate	224.002
Std Err Mean	0.0155	df	15
upper 95% mean	224.0349	Std Dev	0.0618
lower 95% mean	223.9690	Sigma given	0.004
N	16.0000		z Test
		Test Statistic	1.9375
		Prob > |z|	0.0527
		Prob > z	0.0263
		Prob < z	0.9737

(a) Should she be concerned? Include the hypotheses for this test, the test statistic, the p-value, and your conclusion.
(b) Give a 95% confidence interval for the true mean. Could this confidence interval be used to test the hypothesis in (a)? Justify your answer.

(See the solution on page 112.)

17. Forty-four 2-litre cartons of milk from a local dairy were randomly selected and the time in days it took until the milk went sour was recorded. Health code requirements are that a carton of milk must last an average of at least 40 days before going sour. At right are the results of a *JMP™* analysis on whether this dairy is meeting code.

(a) A histogram of the sample is strongly left-skewed. Is this a concern?

(b) What are the hypotheses and conclusion, including the p-value.

Test Mean=value		
Hypothesized Value		40
Actual Estimate		35.0909
df		43
Std Dev		11.1893
	t Test	
Test Statistic	-2.9102	
Prob > |t|	0.0057	
Prob > t	0.9971	
Prob < t	0.0029	

(See the solution on page 114.)

18. Coordination tests conducted on 22 subjects have been input to *JMP™* with the output at right obtained. Which of the following statements is correct?

Test Mean=value		
Hypothesized Value		80
Actual Estimate		75.238
df		21
Std Dev		7.675
	t Test	
Test Statistic	–2.2341	
Prob > |t|	0.0301	
Prob > t	0.9850	
Prob < t	0.0150	

(A) The P-value for testing H_0: $\mu = 75.238$ vs. H_a: $\mu \neq 75.238$ is 0.0301.

(B) The P-value for testing H_0: $\mu = 75.238$ vs. H_a: $\mu < 75.238$ is 0.0150.

(C) The P-value for testing H_0: $\mu = 75.238$ vs. H_a: $\mu > 75.238$ is 0.9850.

(D) One can reject the null hypothesis H_0: $\mu = 80$ at the 99% significance level when the alternative hypothesis is H_a: $\mu > 80$.

(E) One cannot reject the null hypothesis H_0: $\mu = 80$ at the 2.5% significance level when the alternative hypothesis is H_a: $\mu < 80$.

(See the solution on page 115.)

SUMMARY OF KEY CONCEPTS IN LESSON 3

- Memorize the definitions of Type I and Type II errors.
  - **TYPE I ERROR: Rejecting H_0 when H_0 is true.**
  - **TYPE II ERROR: Not rejecting H_0 when H_0 is false.**
- The probability of Type I error is α, the level of significance. The probability of Type II error is β. There is no direct connection between α and β.
- If a researcher believes one error would have much more serious consequences than the other, they should set up their null and alternative hypotheses in such a way that the Type I error would be the more serious one. We have much more direct control over the probability of a Type I error since it equals α, the level of significance that we choose ourselves.
- **The power of a test is the *probability* we will *correctly* reject H_0.**
- **Power = 1 – β.** Put another way, power and β add up to 100%, so once we know one, we know the other.
- **The "α/β/Power" Chain**
  - **If $\alpha\downarrow$, then $\beta\uparrow$, power $\downarrow$.**
  - **If $\alpha\uparrow$, then $\beta\downarrow$, power $\uparrow$.**
- **The "*n*/β/Power" Chain**
  - **If $n\uparrow$, then $\beta\downarrow$, power $\uparrow$.**
  - **If $n\downarrow$, then $\beta\uparrow$, power $\downarrow$.**
- In order to compute α, β, or power for a hypothesis test for the mean, μ, we must first establish the **$\bar{x}$ decision rule**.
  - The formula to compute the critical $\bar{x}$ value is $\bar{x}^* = z^* \frac{\sigma}{\sqrt{n}} + \mu_0$.
  - Study the steps to determine the $\bar{x}$ decision rule on page 132.
- **We must be given an alternative mean, μ_a, in order to compute β or power** for a hypothesis test for the mean, μ.
- The easiest way to compute α, β, or power is to construct an **"α/β Table"**. Study the steps to construct an "α/β Table" on page 138.
- The farther μ_a gets from μ_0, in the appropriate direction, the higher the power will be.

LECTURE PROBLEMS FOR LESSON 3

For your convenience, here are the 9 questions I used as examples in this lesson. Do not make any marks or notes on these questions below. Especially, do not circle the correct choice in the multiple choice questions. You want to keep these questions untouched, so that you can look back at them without any hints. Instead, make any necessary notes, highlights, etc. in the lecture part above.

1. For the table below, indicate which are correct decisions and which are errors. If they are errors, what type?

		True Situation	
		H_0 is true	H_0 is false
Decision	Reject H_0		
	Not to reject H_0		

(See the solution on page 129.)

2. In testing a hypothesis you decide to lower the level of significance from 5% to 1%. Consider the following statements:

(I) The probability of Type I error has decreased from .05 to .01.
(II) The probability of Type II error has increased from .95 to .99.
(III) The power of the test has increased from .95 to .99.
(IV) The probability of Type II error has increased, but the amount cannot be determined.
(V) The probability of Type I error has decreased, but the amount cannot be determined.

(A) only **(I)** is true
(B) only **(I)** and **(II)** are true
(C) only **(I)** and **(III)** are true
(D) only **(I)** and **(IV)** are true
(E) only **(II)** and **(V)** are true

(See the solution on page 130.)

3. A researcher believes the average female executive at any company with more than 500 employees is making less than the 100 thousand dollars a year the average male executive makes. Consequently, she is going to test the hypotheses H_0: $\mu = 100$ thousand dollars vs. H_a: $\mu < 100$ thousand dollars. From previous research, it is known the distribution of executive salaries is approximately normal with $\sigma = 20$ thousand dollars. Determine the $\bar{x}$ decision rule in the following cases:

(a) She will select a random sample of 16 female executives and use a 10% level of significance.

(See the solution on page 134.)

(b) She will select a random sample of 16 female executives and use a 5% level of significance.

(See the solution on page 134.)

(c) She will select a random sample of 64 female executives and use a 5% level of significance.

(See the solution on page 135.)

(d) She will select a random sample of 64 female executives and use a 2% level of significance.

(See the solution on page 135.)

4. You have an SRS (simple random sample) of size $n = 16$ from a normal distribution with $\sigma = 6$. You wish to test H_0: $\mu = 12$ vs. H_a: $\mu > 12$, and you decide to reject H_0 if $\bar{x} > 15$.

(a) What is the probability of a Type I error?

(A) 0.0228 **(B)** 0.05 **(C)** 0.7486 **(D)** 0.4987 **(E)** 0.7734

(See the solution on page 141.)

(b) If $\mu = 16$ in fact, what is the probability of a Type II error?

(A) 0.0013 **(B)** 0.7266 **(C)** 0.7486 **(D)** 0.2734 **(E)** 0.2514

(See the solution on page 142.)

(c) If $\mu = 16$ in fact, what is the power of the test?

(A) 0.9987 **(B)** 0.7266 **(C)** 0.7486 **(D)** 0.2734 **(E)** 0.2514

(See the solution on page 142.)

5. You are examining a normal population with $\sigma = 10$. You wish to test H_0: $\mu = 30$ vs. H_a: $\mu \neq 30$ by taking a random sample of size 25. You decide to reject H_0 if $\bar{x} < 27$ or $\bar{x} > 33$.

(a) What is the probability of a Type I error?

(A) 0.0668 **(B)** 0.0548 **(C)** 0.1336 **(D)** 0.1096 **(E)** 0.05

(See the solution on page 144.)

(b) If $\mu = 28$ in fact, what is the power of the test?

(A) 0.3147 **(B)** 0.9332 **(C)** 0.05 **(D)** 0.6853 **(E)** 0.8664

(See the solution on page 145.)

(c) If $\mu = 25$ in fact, what is the power of the test?

(A) 0.1587 **(B)** 0.8849 **(C)** 0.8907 **(D)** 0.1093 **(E)** 0.8413

(See the solution on page 147.)

6. A sample of size 15 is selected from a normal population with variance 100. The null hypothesis H_0: $\mu = 77$ is tested against the alternative hypothesis H_a: $\mu > 77$ at the 5% level.

(a) What is the probability of a Type II error if μ is actually 83?

(A) 0.2517 **(B)** 0.7483 **(C)** 0.2483 **(D)** 0.7517 **(E)** 0.0500

(b) What is the probability of a Type I error?

(A) 0.2517 **(B)** 0.7483 **(C)** 0.2483 **(D)** 0.7517 **(E)** 0.0500

(See the solution on page 151.)

7. A researcher believes a company's claim its batteries last an average of at least 100 hours is bogus. He intends to take a random sample of 64 batteries and will reject their claim if $z < -2$. He assumes the population has a standard deviation of 10 hours.

(a) What is a Type I error in this context?

(See the solution on page 153.)

(b) What is a Type II error in this context?

(See the solution on page 153.)

(c) What level of significance is the researcher using?

(See the solution on page 155.)

(d) If the true mean is 90 hours, then what is the power of this test?

(See the solution on page 156.)

(e) If the true mean is 90 hours, is this a reliable test?

(See the solution on page 156.)

(f) If the true mean is 98 hours, is this a reliable test?

(See the solution on page 157.)

8. Bottles of a popular cola are supposed to contain 343 ml on average with a standard deviation of 3 ml. An inspector is concerned that the company is underfilling the bottles and, assuming the distribution is normal, plans to select 6 bottles at random to test this concern at the 5% level of significance. If the true mean fill is 342 ml:

(a) What is the power of this test?

(A) 0.2251 **(B)** 0.2033 **(C)** 0.7749 **(D)** 0.7967 **(E)** 0.8708

(See the solution on page 159.)

(b) The power of this test would increase if:

(A) The inspector would take more samples.
(B) The true mean was 340 ml.
(C) The inspector used a higher level of significance.
(D) Only (A) and (C) are true.
(E) (A), (B) and (C) are all true.

(See the solution on page 160.)

9. A breakfast cereal claims to have an average of 2.7 grams of dietary fibre per 30 gram serving. A researcher, interested in seeing if the actual amount of fibre is different, assumes the population is normal with a standard deviation of 0.3 grams. Using a 5% significance level, she will measure the dietary fibre in 20 randomly selected 30 gram servings. If the true average is actually 2.5 grams, what is the probability of a Type II error?

(A) 0.8849 **(B)** 0.1539 **(C)** 0.8461 **(D)** 0.1151 **(E)** 0.8708

(See the solution on page 162.)

SUMMARY OF KEY CONCEPTS IN LESSON 4

- Memorize the properties of mean and variance.
  - $\mu_{X+Y} = \mu_X + \mu_Y$
  - $\mu_{X-Y} = \mu_X - \mu_Y$
- If the two random variables are independent:
  - $\sigma^2_{X+Y} = \sigma^2_X + \sigma^2_Y$
  - $\sigma^2_{X-Y} = \sigma^2_X + \sigma^2_Y$
- If the two random variables are dependent with correlation coefficient ρ:
  - $\sigma^2_{X+Y} = \sigma^2_X + \sigma^2_Y + 2\rho\sigma_X\sigma_Y$
  - $\sigma^2_{X-Y} = \sigma^2_X + \sigma^2_Y - 2\rho\sigma_X\sigma_Y$
- A **matched pairs experiment** applies both treatment *A* and treatment *B* to each unit, allowing you to compute the difference in treatment scores for each pair. A **two-sample experiment** selects one random sample of size n_1 to receive treatment *A* and a second independent random sample of size n_2 to receive treatment *B*. Make sure you can tell the difference between matched pairs data and two-sample data.
- **In the matched pairs *t* test**, there is only one set of data because we have computed the differences between the scores for each pair. We test H_0: $\mu = 0$ where μ is the mean of the differences, using $\bar{x}$, the sample mean of our differences, and *s*, the standard deviation of the differences.
- **In a two-sample *t* test** we have two of everything. Two sample sizes, n_1 and n_2, two sample means, $\bar{x}_1$ and $\bar{x}_2$, and two sample standard deviations, s_1 and s_2.
  - **A confidence interval for $\mu_1 - \mu_2$ is:** $(\bar{x}_1 - \bar{x}_2) \pm t^* SE(\bar{x}_1 - \bar{x}_2)$
  - **To test H_0: $\mu_1 = \mu_2$, the test statistic is:** $t = \dfrac{\bar{x}_1 - \bar{x}_2}{SE(\bar{x}_1 - \bar{x}_2)}$
- First we must decide whether to assume $\sigma_1^2 = \sigma_2^2$ or not. If they have not specifically told us what to assume, we will use the **Rule of Thumb**.
  - **If** $\dfrac{\text{larger value of } s_1 \text{ and } s_2}{\text{smaller value of } s_1 \text{ and } s_2} < 2$, **then assume** $\sigma_1^2 = \sigma_2^2$.

- The formula sheet we will be given on our exam shows us the formulas to use for $SE(\bar{x}_1 - \bar{x}_2)$ and the degrees of freedom depending on whether we are assuming the variances are equal or not.
- If we are assuming $\sigma_1^2 = \sigma_2^2$, we will use the **pooled two-sample *t* test**; if not, we will use the **generalized two-sample *t* test**.
- Both the pooled and generalized methods require that we have two simple random samples that are selected independently of each other (two independent, random samples).
- **The generalized method is robust.** As long as the total sample size is large we will get reliable results. We would only require normal populations if $n_1 + n_2 < 15$. The generalized method is reliable in most cases if $n_1 + n_2 \geq 15$. If one or both of the populations is strongly skewed or has outliers, then we want $n_1 + n_2 \geq 40$.
- **The pooled method is <u>not</u> robust.** It is only reliable if both populations are normally distributed. Of course, both populations must have the same variance as well (if $\sigma_1^2 \neq \sigma_2^2$, we shouldn't be using the pooled method).
- **The generalized method does not actually have a *t* distribution.** However, its results are very similar to a *t* distribution with degrees of freedom approximately the same as that given by that horrid formula on our formula sheet (thank goodness we do not have to memorize this formula):
  - $$\text{df} = \frac{\left(\frac{s_1^2}{n_1} + \frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1 - 1}\left(\frac{s_1^2}{n_1}\right)^2 + \frac{1}{n_2 - 1}\left(\frac{s_2^2}{n_2}\right)^2}$$
  - This formula invariably results in a decimal answer (not a whole number). Degrees of freedom must be a whole number. Use what I call ***The Price is Right Rule***. Use the closest whole number degrees of freedom without going over. For example, if you compute df = 23.99, you would say the df = 23.
  - **The pooled method has an exact *t* distribution with df = $n_1 + n_2 - 2$.**
- We will also use *The Price is Right Rule* anytime Table D does not happen to have the degrees of freedom we require. Make do with the closest degrees of freedom that is on Table D without going over.

LECTURE PROBLEMS FOR LESSON 4

For your convenience, here are the 14 questions I used as examples in this lesson. Do not make any marks or notes on these questions below. Especially, do not circle the correct choice in the multiple choice questions. You want to keep these questions untouched, so that you can look back at them without any hints. Instead, make any necessary notes, highlights, etc. in the lecture part above.

1. The mean annual income of men is 40 thousand dollars with a standard deviation of 7 thousand dollars. The mean annual income of women is 35 thousand dollars with a standard deviation of 12 thousand dollars.

(a) It is fair to assume that, if we randomly selected one man and one woman, their salaries would be independent. Keeping that in mind, what is the mean and standard deviation of their total income?

(See the solution on page 173.)

(b) What is the mean and standard deviation of the difference in their incomes (Men – Women)?

(See the solution on page 173.)

(c) Assume the mean and standard deviations for the income of married men and women are the same as given above. However, a husband's income is not independent of his wife's. The correlation between a husband's income and his wife's is .4. What is the mean and standard deviation of the total income of a husband and wife?

(See the solution on page 174.)

2. The mean cost of textbooks is $110 and the standard deviation is $25. A typical student has to purchase five textbooks in a given term. Assuming the price of each textbook is independent, what is the mean and standard deviation, respectively, of the total cost of five textbooks for a given term.

(A) $550 and $55.90 **(B)** $550 and $125 **(C)** $110 and $25
(D) $550 and $5 **(E)** $22 and $5

(See the solution on page 175.)

3. To determine whether an epilepsy drug is useful in treating children with severe learning problems, 5 children with a history of learning and behavioural problems are recruited. Each child was given a placebo for 3 weeks and the epilepsy drug for the other 3 weeks. After each 3 week period, all 5 children were given an IQ test. The results below were recorded. Set up the appropriate hypothesis test to determine if the drug improves IQ at the .10 level of significance. What are your assumptions and conclusions?

Child	1	2	3	4	5
IQ after Placebo	97	106	106	95	126
IQ after Drug	113	113	101	119	126

(See the solution on page 183.)

4. Two different methods to determine the fat content in meat are being compared. Twenty-five pieces of meat are randomly selected. Each piece of meat is then cut in half and one half is randomly selected to be analyzed using Method A, the other half is analyzed using method B, the difference in the two analyses is then recorded. The mean of these 25 differences is –1.02 with a standard deviation of 1.76. We are investigating whether the methods give different results at the 5% level of significance.

(a) What experimental design is being used and what would be the appropriate hypotheses?

(A) Completely randomized design; H_0: $\mu = -1.02$ vs. H_a: $\mu \neq -1.02$.

(B) Block design; H_0: $\mu = -1.02$ vs. H_a: $\mu \neq -1.02$.

(C) Matched pairs design; H_0: $\mu = -1.02$ vs. H_a: $\mu \neq -1.02$.

(D) Completely randomized design; H_0: $\mu = 0$ vs. H_a: $\mu \neq 0$.

(E) Matched pairs design; H_0: $\mu = 0$ vs. H_a: $\mu \neq 0$.

(See the solution on page 184.)

(b) The rejection region for the test is:

(A) $t > 1.711$ **(B)** $|t| > 2.064$ **(C)** $|t| < 2.064$

(D) $t < -1.711$ **(E)** $|z| > 1.96$

(See the solution on page 185.)

(c) The P-value is

(A) between .005 and .0025 **(B)** between .01 and .005

(C) between .025 and .01 **(D)** .0019 **(E)** .0038

(See the solution on page 186.)

5. Twelve samples of brand *A* produced a mean of 16 and a variance of 36. Fifteen samples of brand *B* have a mean of 18 and a variance of 33. The pooled sample variance is

(A) 34.32 **(B)** 294 **(C)** 1180 **(D)** 34.33 **(E)** 5.86

(See the solution on page 201.)

6. We are interested in comparing the annual salaries of registered nurses with at least 5 years of experience in two provinces. A random sample of 23 nurses in Manitoba fitting the criteria produced a mean salary of 36 thousand dollars per year with a standard deviation of 5 thousand dollars. A random sample of 21 nurses in Ontario fitting the criteria produced a mean salary of 40 thousand dollars with a standard deviation of 8 thousand dollars.

(a) Making appropriate assumptions, is there evidence the mean salary of registered nurses with at least 5 years experience differs in these two provinces at the 4% level of significance? Define your notation, and include your hypotheses, degrees of freedom, critical value, and test statistic in your answer.

(See the solution on page 205.)

(b) What is the *P*-value for the test you conducted in (a)?

(See the solution on page 206.)

(c) Construct a 96% confidence interval for the difference in the mean salary of registered nurses with at least 5 years experience in these two provinces. Could you use this confidence interval to test the hypothesis in (a)? Explain.

(See the solution on page 207.)

7. A random sample of 24 thirteen-year-old boys reveals they spend an average of 8 hours per week playing video games with a standard deviation of 3 hours; whereas, a random sample of 24 thirteen-year-old girls plays video games for an average of 2 hours per week with a standard deviation of 1 hour.

(a) Making appropriate assumptions, is this strong evidence thirteen-year-old boys play video games more often than girls do, on average, at the 5% level of significance? Define your notation, and include your hypotheses, degrees of freedom, critical value, and test statistic in your answer.

(See the solution on page 211.)

(b) What is the *P*-value for the test you conducted in (a)?

(See the solution on page 211.)

(c) Construct a 95% confidence interval for the difference in the average time spent playing video games by the two sexes. Could you use this confidence interval to test the hypothesis in (a)? Explain.

(See the solution on page 212.)

8.

	Number of Children	Mean Score	Variance
Anglo-American	12	74	64
Mexican-American	10	67	100

A reading test is given to an elementary school class that consists of Anglo-American and Mexican-American children to see if the latter have weaker reading skills. Using the results shown above, and assuming common variance, the value of the test statistic is:

(A) $\dfrac{74-67}{\sqrt{\dfrac{64}{12}+\dfrac{100}{10}}}$

(B) $\dfrac{74-67}{\sqrt{\dfrac{64^2}{12}+\dfrac{100^2}{10}}}$

(C) $\dfrac{74-67}{\sqrt{\left(\dfrac{(11)(64)+(9)(100)}{20}\right)\left(\dfrac{1}{12}+\dfrac{1}{10}\right)}}$

(D) $\dfrac{74-67}{\sqrt{\left(\dfrac{(11)(64)+(9)(100)}{22}\right)\left(\dfrac{1}{12}+\dfrac{1}{10}\right)}}$

(E) $\dfrac{74-67}{\sqrt{\left(\dfrac{(11)(64^2)+(9)(100^2)}{20}\right)\left(\dfrac{1}{12}+\dfrac{1}{10}\right)}}$

(See the solution on page 214.)

9. We are comparing the lifespan of two brands of light bulbs. The information at right is collected from random samples. The standard error for the difference in the two means would be:

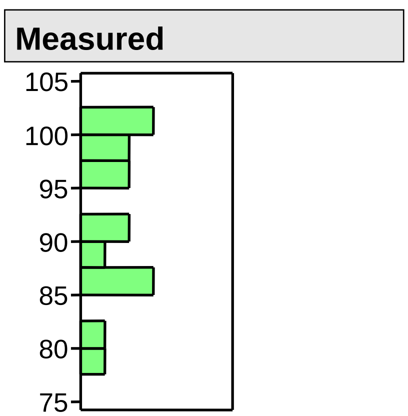
(A) $\sqrt{\dfrac{120}{41}+\dfrac{40}{51}}$ **(B)** $\sqrt{\dfrac{120^2}{41}+\dfrac{40^2}{51}}$ **(C)** $\sqrt{\dfrac{1190^2}{41}+\dfrac{1160^2}{51}}$

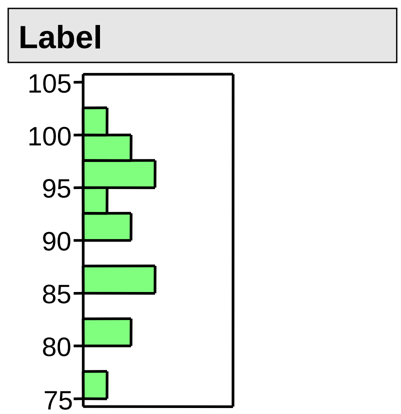
(D) $\sqrt{\dfrac{1190}{41}+\dfrac{1160}{51}}$ **(E)** $\sqrt{\dfrac{(40)(120)^2+(50)(40)^2}{90}\left(\dfrac{1}{41}+\dfrac{1}{51}\right)}$

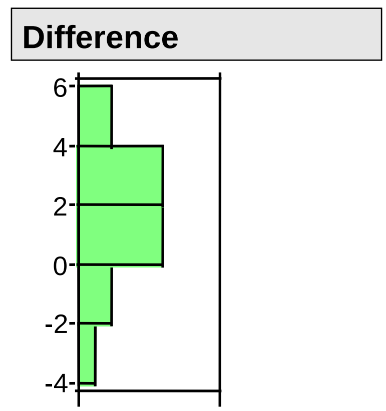
(See the solution on page 215.)

10. A researcher wishes to determine if diet food labels tend to under-report the number of calories that the item contains. To investigate this, the researcher obtains a simple random sample of 15 diet food items. For each item, she measures the actual calorie content ("Measured") and compares it with the number of calories stated on the label ("Label"). From the information given, answer the questions below.

Food item	1	2	3	4	5	6	7	8
Measured	82.0	88.0	99.5	96.5	91.4	100.0	86.5	97.6
Label	85.0	90.0	101.0	96.0	90.5	99.0	85.0	96.0
Difference	–3.0	–2.0	–1.5	0.5	0.9	1.0	1.5	1.6
Food item	**9**	**10**	**11**	**12**	**13**	**14**	**15**	
Measured	100.0	79.5	96.5	100.5	85.6	90.0	85.0	
Label	98.0	77.0	94.0	97.0	82.0	86.0	80.0	
Difference	2.0	2.5	2.5	3.5	3.6	4.0	5.0	







Moments	
Mean	91.90667
Std Dev	7.19647
Std Err Mean	1.85812
upper 95% Mean	95.89193
lower 95% Mean	87.92140
N	15.00000

Moments	
Mean	90.43333
Std Dev	7.55661
Std Err Mean	1.95111
upper 95% Mean	94.61804
lower 95% Mean	86.24863
N	15.00000

Moments	
Mean	1.473333
Std Dev	2.272024
Std Err Mean	0.586634
upper 95% Mean	2.731536
lower 95% Mean	0.215131
N	15.00000

(a) What would be the appropriate test to use in this problem and, with reference to the histograms provided, is the procedure valid?

(See the solution on page 218.)

(b) State the appropriate hypotheses to conduct this test defining your notation clearly.

(See the solution on page 218.)

(c) What conclusion can the researcher make at the 10% level of significance?

(See the solution on page 219.)

(d) State the Type I and Type II errors in this context.

(See the solution on page 219.)

11. Random samples of 11 defensive lineman and 8 offensive linemen who play college football in the US were selected and their weight (in pounds) was measured.

Oneway Analysis of Weight By Position

Oneway Anova

Summary of Fit

Rsquare	0.535643
Adj Rsquare	0.508328
Root Mean Square Error	21.15167
Mean of Response	281.0526
Observations (or Sum Wgts)	19

t-Test

Assuming equal variances

	Difference	t-Test	DF	Prob > |t|
Estimate	-43.523	-4.428	17	0.0004
Std Error	9.828			
Lower 95%	-64.259			
Upper 95%	-22.787			

UnEqual Variances

	Difference	t-Test	DF	Prob > |t|
Estimate	-43.523	-4.179	11.7952	0.0013
Std Error	10.415			
Lower 95%	-64.978			
Upper 95%	-22.067			

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
Position	1	8773.266	8773.27	19.6097	0.0004
Error	17	7605.682	447.39		
C. Total	18	16378.947			

Means for Oneway Anova

Level	Number	Mean	Std Error	Lower 95%	Upper 95%
defense	11	262.727	6.3775	249.27	276.18
offense	8	306.250	7.4782	290.47	322.03

Std Error uses a pooled estimate of error variance

Means and Std Deviations

Level	Number	Mean	Std Dev	Std Err Mean	Lower 95%	Upper 95%
defense	11	262.727	17.6584	5.3242	250.86	274.59
offense	8	306.250	25.3194	8.9518	285.08	327.42

(a) According to the *JMP*™ printout: At the 5% significance level, is there a difference in the mean weight of defensive and offensive linemen? Be sure to state the assumptions, hypotheses, degrees of freedom, test statistic and *P*-value.

(See the solution on page 222.)

(b) What is a 95% confidence interval for the difference in mean weight between defensive and offensive lineman? (Use the *JMP*™ printout.)

(See the solution on page 223.)

12. Do smokers spend more time "away from their desk"? The *JMP*™ analysis below looks at the average number of minutes insurance clerks in a large "smoke-free" firm spent away from their desk on a given day. Twenty-one randomly selected smokers and twenty-one randomly selected nonsmokers were monitored. Use this printout to answer the following questions.

Oneway Analysis of Time Away By Smoking Preference

Oneway Anova

Summary of Fit

Rsquare	0.103782
Adj Rsquare	0.081376
Root Mean Square Error	3.513081
Mean of Response	18.72381
Observations (or Sum Wgts)	42

t-Test

Assuming equal variances

	Difference	t-Test	DF	Prob > |t|
Estimate	-2.3333	-2.152	40	0.0375
Std Error	1.0842			
Lower 95%	-4.5245			
Upper 95%	-0.1422			

UnEqual Variances

	Difference	t-Test	DF	Prob > |t|
Estimate	-2.3333	-2.152	26.9173	0.0405
Std Error	1.0842			
Lower 95%	-4.5582			
Upper 95%	-0.1085			

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
Time Away	1	57.16667	57.16667	4.6320	0.0375
Error	40	493.66952	12.3417		
C. Total	41	550.83619			

Means for Oneway Anova

Level	Number	Mean	Std Error	Lower 95%	Upper 95%
Nonsmokers	21	17.5571	0.76662	16.008	19.107
Smokers	21	19.8905	0.76662	18.341	21.440

Std Error uses a pooled estimate of error variance

Means and Std Deviations

Level	Number	Mean	Std Dev	Std Err Mean	Lower 95%	Upper 95%
Nonsmokers	21	17.5571	1.93328	0.42188	16.677	18.437
Smokers	21	19.8905	4.57667	0.99871	17.807	21.974

(a) At the 5% significance level, is there evidence smokers spend more time away from their desk? Be sure to state the assumptions, hypotheses, degrees of freedom, test statistic and *P*-value.

(b) What is a 95% confidence interval for the difference in mean time smokers and nonsmokers spend away from their desk?

(See the solution on page 227.)

13. Two laboratory procedures for determining the amylase level in human body fluids are compared. The "New" method is less expensive than the "Standard" method, but may give different results. To test the procedures, samples from 20 subjects are randomly assigned 10 to each lab procedure with the following results (in units per millilitre). A variable called "New minus Standard" was also defined. The *JMP*™ analysis follows.

Standard	38	53	58	53	75	58	59	46	69	59
New	46	57	73	60	86	37	65	58	85	74
New minus Standard	8	4	15	7	11	-21	6	8	16	15

New Moments	
Mean	64.10000
Std Dev	15.84964
Std Err Mean	5.01210
upper 95% Mean	75.43825
lower 95% Mean	52.76175
N	10.00000

Standard Moments	
Mean	56.80000
Std Dev	10.49656
Std Err Mean	3.31930
upper 95% Mean	64.30885
lower 95% Mean	49.29115
N	10.00000

New minus Standard Moments	
Mean	7.300000
Std Dev	10.770845
Std Err Mean	3.406040
upper 95% Mean	15.005064
lower 95% Mean	-0.405064
N	10.000000

(a) What kind of test would be appropriate in order to determine if there is any difference in the methods. Use the given information to help in your explanation, and state the hypotheses and assumptions.

(b) What conclusion can we make about the methods at the 5% level of significance?

(c) Construct a 99% confidence interval for the difference between the means.

(d) Suggest an improved experimental design for this problem.

(See the solution on page 230.)

14. The data below shows the annual salary (in thousands of dollars) of 6 randomly selected Finance majors and 6 randomly selected Marketing majors 10 years after they graduated from the Faculty of Management at U of M.

Finance	81.6	91.7	75.4	100.2	93.0	106.5
Marketing	85.2	68.3	39.2	81.9	77.1	75.3

(a) Stating all appropriate assumptions, test the hypothesis that the mean salary of Finance majors is higher than that of Marketing majors at the 5% level of significance.

(b) What would be the Type I error and Type II error in this context?

(See the solution on page 234.)

SUMMARY OF KEY CONCEPTS IN LESSON 5

- In general, the One-Way Analysis of Variance method will test the null hypothesis that the means of *I* samples are all equal.
  - H_0: $\mu_1 = \mu_2 = \mu_3 \ldots = \mu_I$ versus H_a: not all the means are equal
- The formulas for the group and error degrees of freedom (also called the numerator and denominator degrees of freedom, respectively) are:
  - $DFG = I - 1$
  - $DFE = N - I$
- The formulas required to conduct ANOVA are:
  - The **overall mean** = $\bar{\bar{x}} = \dfrac{\sum n_i \bar{x}_i}{N} = \dfrac{n_1\bar{x}_1 + n_2\bar{x}_2 + n_3\bar{x}_3 + \ldots + n_I\bar{x}_I}{N}$ (memorize this!)
  - $MSG = \dfrac{SSG}{DFG} = \dfrac{\sum n_i\left(\bar{x}_i - \bar{\bar{x}}\right)^2}{I-1} = \dfrac{n_1\left(\bar{x}_1 - \bar{\bar{x}}\right)^2 + n_2\left(\bar{x}_2 - \bar{\bar{x}}\right)^2 + \ldots + n_I\left(\bar{x}_I - \bar{\bar{x}}\right)^2}{I-1}$

    (The formula sheet you are given on the exam gives you the formula for *SSG*, so you need only remember that $MSG = \dfrac{SSG}{DFG}$.)
  - $MSE = \dfrac{SSE}{DFE} = \dfrac{\sum (n_i - 1)s_i^2}{N-I} = \dfrac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2 + \ldots + (n_I - 1)s_I^2}{N-I}$

    ($\boldsymbol{s_p^2 = MSE}$; the formula sheet gives you the formula for the pooled sample variance for a two-sample problem; *MSE* simply extends that pooled sample variance for two, three, four, or more samples).
  - The formula for the *F* test statistic is $\boldsymbol{F = \dfrac{MSG}{MSE}}$ with df = *DFG*, *DFE*.
- One-Way Analysis of Variance is comparing the **variance among** the means of the groups (*MSG*) to the **variance within** the samples themselves (*MSE*).
- **The ANOVA *F* test is <u>not</u> robust.** For the test to be reliable:
  - We must have independent, simple random samples.
  - All the populations of interest must be normally distributed.
  - The populations must have the same variance.

- **Rule of Thumb:** If $\frac{\text{largest value of } s}{\text{smallest value of } s} < 2$, then assume $\sigma_1^2 = \sigma_2^2 = \sigma_3^2 = \ldots = \sigma_I^2$.

- Make sure you are able to fill in an ANOVA Table given only a couple of entries in the table as I teach you on page 265. That is a very common task on exams.

- **The coefficient of determination** $= R^2 = \frac{SSG}{SST}$.
  - R^2 will always have a value between 0 and 1 and can be expressed as a percentage. R^2 is what percentage of the total variation in the sample (SST) is explained by the variation among the group means themselves (SSG).

- Be careful, if you are ever asked to make a confidence interval for a mean or a confidence interval for the difference between two means in a problem where ANOVA has been performed.
  - In any problem where an ANOVA is conducted, you are committed to the assumption of common variance. Consequently, any confidence interval you make for a mean or difference between two means must use the pooled sample variance, $s_p^2 = MSE$ to compute the appropriate standard error. Additionally, the critical value, t^*, in the confidence interval will have df $= DFE = N - I$.

- There are two key things a **normal quantile plot** can tell us:
  - If a normal quantile plot looks linear, it is fair to assume the sample is from a normal population. If the plot is nonlinear, or has outliers, it is doubtful the population is normal.
  - When we have more than one sample, we graph each sample on the same normal quantile plot. Assuming the data looks linear, we will have one line for each sample. If all the lines are parallel, it is fair to assume the normal populations have the same variance.

- If we are testing H_0: $\mu_1 = \mu_2$ vs. H_a: $\mu_1 \neq \mu_2$ for two normal populations where $\sigma_1^2 = \sigma_2^2$, it is valid to use the ***pooled* two-sample *t* test or the ANOVA *F* test**.
  - Both tests are using the exact same estimate of the variance ($s_p^2 = MSE$). Both tests will have the same P-value and lead to the same conclusion.
  - In fact, the test statistics t and F are related in that $\boldsymbol{t^2 = F}$.

LECTURE PROBLEMS FOR LESSON 5

For your convenience, here are the 7 questions I used as examples in this lesson. Do not make any marks or notes on these questions below. Especially, do not circle the correct choice in the multiple choice questions. You want to keep these questions untouched, so that you can look back at them without any hints. Instead, make any necessary notes, highlights, etc. in the lecture part above.

1. At what age do babies learn to crawl? Does it take longer to learn in the winter when babies are often bundled in clothes that restrict their movement? Data were collected from parents who brought their babies into the University of Denver Infant Study Center to participate in one of a number of experiments between 1988 and 1991. Parents reported the birth month and the age at which their child was first able to creep or crawl a distance of four feet within one minute. The resulting data were grouped by month of birth. The data are for January, May, and September. Crawling age is given in weeks.

Birth month	Average crawling age	SD	n
January	29.84	7.08	32
May	28.58	8.07	27
September	33.83	6.93	38

(a) Perform an Analysis of Variance test on this data at the 5% level of significance. Be sure to state your hypotheses, degrees of freedom, critical value, test statistic, and conclusion.

(See the solution on page 257.)

(b) What is the P-value for this test?

(See the solution on page 259.)

(c) What assumptions are you making?

(See the solution on page 260.)

2. An employer keeps track of the productivity of his furniture factory. Productivity is measured in terms of wholesale value, in hundreds of dollars, of the furniture produced by his employees and is recorded daily for five consecutive weeks. There are only four pieces of data for Monday because the factory was closed for a public holiday during this period.

Daily Productivity (hundreds of dollars)

	Monday	**Tuesday**	**Wednesday**	**Thursday**	**Friday**
Week 1	143	162	160	138	110
Week 2	128	136	132	168	130
Week 3		144	180	120	135
Week 4	117	158	160	152	142
Week 5	132	144	168	148	126

Assume that productivity on any weekday is normally distributed with the same variance and that productivity on one day does not influence productivity on another day.

Is there significant evidence at the 2.5% level of significance that productivity differs on different days of the week? Include a *P*-value in your conclusion.

(See the solution on page 264.)

3. In a completely randomized experiment, five different brands of cigarettes were tested to see what their levels of nicotine were. Six cigarettes from Brand A were tested; five of Brand B; seven of Brand C; eight of Brand D; and four of Brand E. Complete the following ANOVA table to determine the *F*-ratio.

Analysis of Variance

Source	**DF**	**Sum of Squares**	**Mean Square**	**F Ratio**
Brand		200		
Error				
C. Total		296		

Is there any difference in the mean nicotine content of the five brands? Use a 1% level of significance, and include the *P*-value in your conclusion.

(See the solution on page 269.)

4. An ANOVA table was set up to compare the effectiveness of three different insecticides in controlling a species of parasitic beetle. A partial output is below.

Analysis of Variance				
Source	**DF**	**Sum of Squares**	**Mean Square**	**F Ratio**
Insecticide			392.07	
Error				
C. Total	14	845.34		

(a) The null and alternative hypotheses for this problem above are:

(A) H_0: $\mu_1 = \mu_2 = \mu_3$ versus H_a: $\mu_1 \neq \mu_2 \neq \mu_3$

(B) H_0: $\mu_1 = \mu_2 = \mu_3$ versus H_a: not all of μ_1, μ_2, μ_3 are equal

(C) H_0: $\bar{x}_1 = \bar{x}_2 = \bar{x}_3$ versus H_a: $\bar{x}_1 \neq \bar{x}_2 \neq \bar{x}_3$

(D) H_0: $\sigma_1^2 = \sigma_2^2 = \sigma_3^2$ versus H_a: not all the variances are equal

(E) None of the above.

(b) Which of the following must be true for the analysis of variance to be valid?

(A) The samples were chosen randomly.

(B) The samples from each population are independent.

(C) The populations being compared are normally distributed.

(D) The populations have a common variance.

(E) All of the above.

(c) The pooled sample variance is

(A) 784.12 **(B)** 392.07 **(C)** 61.2 **(D)** 12 **(E)** 5.1

(d) The critical value at $\alpha = .05$ and test statistic for this problem are, respectively,

(A) 5.10, 76.9 **(B)** 3.89, 12.8 **(C)** 3.89, 76.9 **(D)** 5.10, 12.8 **(E)** 2.81, 5.10

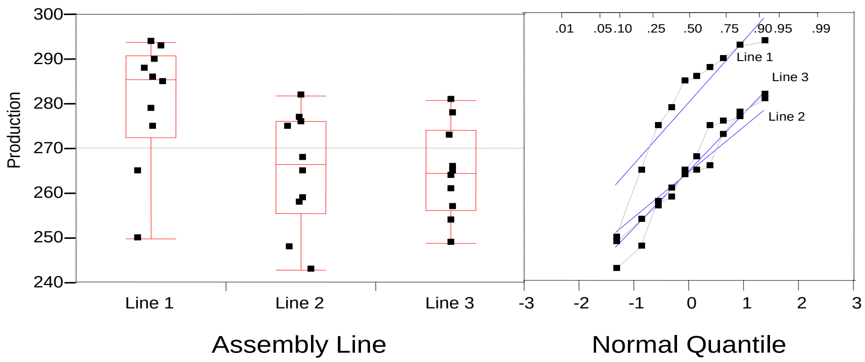
(e) The coefficient of determination for this problem is

(A) 0.143 **(B)** 0.928 **(C)** 0.072 **(D)** 0.078 **(E)** 0.922

(See the solution on page 271.)

5. The manager of an automotive parts factory wished to see if three different methods of assembly line procedures made a difference in overall production. The methods were assigned randomly to three different assembly lines and the amount of parts produced during an 8-hour shift were recorded. The *JMP*™ printout is shown below.

Oneway Analysis of Production By Assembly Line



Oneway Anova

Summary of Fit

Rsquare	0.277082
Adj Rsquare	0.223532
Root Mean Square Error	12.48258
Mean of Response	270.1333
Observations (or Sum Wgts)	30

Analysis of Variance

Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
Assembly Line	2	1612.4667	806.233	5.1743	0.0125
Error	27	4207.0000	155.815		
C. Total	29	5819.4667			

Means for Oneway Anova

Level	Number	Mean	Std Error	Lower 95%	Upper 95%
Line 1	10	280.500	3.9473	272.40	288.60
Line 2	10	265.100	3.9473	257.00	273.20
Line 3	10	264.800	3.9473	256.70	272.90

Std Error uses a pooled estimate of error variance

Means and Std Deviations

Level	Number	Mean	Std Dev	Std Err Mean	Lower 95%	Upper 95%
Line 1	10	280.500	13.8984	4.3951	271.48	289.52
Line 2	10	265.100	12.9996	4.1108	256.67	273.53
Line 3	10	264.800	10.2610	3.2448	258.14	271.46

(a) Use the ANOVA to test the appropriate hypothesis. Be sure to state the hypotheses, test statistic, degrees of freedom, and *P*-value.

(b) Make a 95% confidence interval for the difference in the mean production levels of Line 1 and Line 3.

(c) Make a 95% confidence interval for the mean production level of Line 1.

(d) With reference to the output, is the use of ANOVA justified?

(See the solution on page 276.)

6. Return to question 2 above (the comparison of productivity at a furniture factory for each day of the week).

(a) Construct a 98% confidence interval for the mean productivity on Friday.

(b) Construct a 98% confidence interval for the difference between the mean productivity on Wednesday and the mean productivity on Monday.

(See the solution on page 286.)

7. Random samples of 11 defensive lineman and 8 offensive linemen who play college football in the US were selected and their weight (in lbs) was measured.

Oneway Analysis of Weight By Position

Oneway Anova

t-Test

Assuming equal variances

	Difference	t-Test	DF	Prob > |t|
Estimate	-43.523	-4.428	17	0.0004
Std Error	9.828			
Lower 95%	-64.259			
Upper 95%	-22.787			

UnEqual Variances

	Difference	t-Test	DF	Prob > |t|
Estimate	-43.523	-4.179	11.7952	0.0013
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Source	DF	Sum of Squares	Mean Square	F Ratio	Prob > F
Position	1	8773.266	8773.27	19.6097	0.0004
Error	17	7605.682	447.39		
C. Total	18	16378.947			

Means for Oneway Anova

Level	Number	Mean	Std Error	Lower 95%	Upper 95%
defense	11	262.727	6.3775	249.27	276.18
offense	8	306.250	7.4782	290.47	322.03

Std Error uses a pooled estimate of error variance

Means and Std Deviations

Level	Number	Mean	Std Dev	Std Err Mean	Lower 95%	Upper 95%
defense	11	262.727	17.6584	5.3242	250.86	274.59
offense	8	306.250	25.3194	8.9518	285.08	327.42

(a) Using both t and F, is there a difference in the mean weight of defensive and offensive linemen? Be sure to state the assumptions, hypotheses, test statistic and P-value. What is the relationship between t and F?

(b) Is there significant evidence defensive linemen are lighter than offensive linemen, on average? Include the test statistic and P-value in your answer.

(c) Is there significant evidence defensive linemen are heavier than offensive linemen, on average? Include the test statistic and P-value in your answer.

(See the solution on page 292.)

SUMMARY OF KEY CONCEPTS IN LESSON 6

- For a discrete random variable $\underline{X}$, we know the sum of all the probabilities in the sample space for X is 1. Which is to say, $\sum \boldsymbol{p(x)} = \mathbf{1}$.
  - The mean of $X = \boldsymbol{\mu} = \sum \boldsymbol{x\,p(x)}$.
  - The variance of $X = \boldsymbol{\sigma^2} = \sum \boldsymbol{(x-\mu)^2\,p(x)}$. Personally, I prefer this formula to compute the variance, though: $\boldsymbol{\sigma^2} = \left(\sum \boldsymbol{x^2 p(x)}\right) - \boldsymbol{\mu^2}$.
  - Of course, many of you can use your calculator in Stat mode to compute the mean and standard deviation of a discrete distribution.
  - If two events A and B are independent, $P(A \text{ and } B) = P(A) \times P(B)$.
- If you are ever asked to find an **expected value**, that is just a Stats "code word" for the **mean value**. Whatever happens on average is what you would "expect" to happen.
- **The parameters of a binomial distribution are *n* and *p*.**
- The binomial distribution is a **discrete** distribution where each trial is **independent**. If we have a fixed number of trials n and if the probability of "yes" is the same for each trial, p, the random variable X has a binomial distribution where X = the number of "yeses". We can say $\boldsymbol{X \sim B\ (n, p)}$.
  - If we are given a **percentage**, a **proportion**, or a **fraction** we are given a value of $\boldsymbol{p}$. We will immediately suspect we have a binomial distribution at that point. All we need is a value for n to clinch it.
  - If we are rolling dice, tossing coins, or guessing on a test, we have a binomial distribution where we are expected to know the value of p ourselves. Again, we must have a specific number of trials n or else the problem is not binomial.
- The binomial probability formula is $\boldsymbol{P(X=k) = \binom{n}{k} p^k (1-p)^{n-k}}$. However, we can also use Table C to solve many binomial probability problems.
  - **If your value of *p* is not on Table C, use the complementary value of *p* instead.** The table gives you the probabilities for $X = 0, 1, 2, \ldots n$ for the given p. Those are also the probabilities, respectively, for $X = n, \ldots 2, 1, 0$ for the complementary value of p.

- If X has a **binomial** distribution, then the **mean of X = μ_X = np** and the **standard deviation of X = σ_X = $\sqrt{np(1-p)}$.**
- Our **Rule of Thumb** says: If $N \geq 10n$, and if $np \geq 10$ and $n(1 - p) \geq 10$, then the random variable X in a binomial distribution is approximately normal.
- If we have boxed in an outrageous amount of X values in a binomial probability problem, we can bet our Rule of Thumb will tell us X is approximately normal, so we can use an ***X*-bell curve** to compute the approximate probability.
  - The standardizing formula for the variable X is $z = \dfrac{x - \mu_X}{\sigma_X} = \dfrac{x - np}{\sqrt{np(1-p)}}$.
- The **Poisson distribution**, like the Binomial distribution, is a **discrete** probability distribution.
  - The Poisson distribution has only one parameter, λ.
- Given the random variable X in a Poisson distribution with parameter λ. Which is to say, given $X \sim Poisson(\lambda)$.
  - The **mean of X = μ_X = λ.**
  - The variance of X = σ_X^2 = λ, also.
  - Thus, the standard deviation of X = $\sigma_X = \sqrt{\lambda}$.
- The Poisson probability formula is $P(X = k) = \dfrac{e^{-\lambda}\lambda^k}{k!}$. (You do not have to memorize this formula since it is included on the formula sheet you will be given on exams.)
- Review how to construct an **"α/β Table"** on page 360. Remember, you can compute the probability of Type I error and Type II error, and you can compute power for a variety of hypotheses. All you need is the decision rule and the "truth" according to the null hypothesis, and an "alternative version of the truth" if you are to compute β or power.

LECTURE PROBLEMS FOR LESSON 6

For your convenience, here are the 20 questions I used as examples in this lesson. Do not make any marks or notes on these questions below. Especially, do not circle the correct choice in the multiple choice questions. You want to keep these questions untouched, so that you can look back at them without any hints. Instead, make any necessary notes, highlights, etc. in the lecture part above.

1. Below is a table showing the various prices a particular model of running shoe was sold for at a local sporting goods store. First the shoe was priced at $150, but then it was gradually reduced in price until the remaining stock was finally sold off at a sale price of $15. The table also includes the proportion of the stock that was sold at each price.

Price of the Running Shoe	$150	$125	$100	$75	$50	$15
Proportion Sold	0.05	0.16	k	$2k$	0.19	k

(a) What is the value of k if this is a properly defined probability distribution?
(b) What is the probability that a randomly selected purchaser did not pay full price ($150) for their running shoes?
(c) What is the probability that a randomly selected purchaser paid less than $100 for their running shoes?
(d) If we randomly selected two purchasers, what is the probability that they both paid $75 for the shoes?
(e) If we randomly selected two purchasers, what is the probability that they paid a total of $150 for the shoes?
(f) What was the average price paid for the running shoes?
(g) What was the standard deviation of the price paid for the running shoes?

(See the solution on page 309.)

2. A travel agent is paid a commission of $80 for each all-inclusive holiday package she sells and $60 for each cruise package she sells. Being of a statistical bent, after some research she estimates the following distributions for her daily sales.

Number of all-inclusive holidays sold	0	1	2	3
Probability	.3	.2	.4	.1

Number of cruise packages sold	0	1	2	3
Probability	.3	.1	.5	.1

Assuming her estimates are correct, what will her mean daily income be?

(See the solution on page 314.)

3. Given the random variable X with probability distribution

$$P(X = x) = \frac{cx+1}{16}, \quad x = 0, 1, 2, 3.$$

Decide the value of the constant c and find the expected value and variance of X.

(See the solution on page 316.)

4. Thirty-five percent of the voters in the last election voted Liberal. If you randomly selected ten voters from the last election, what is the probability exactly four of them voted Liberal?

(See the solution on page 324.)

5. A die is rolled seven times.

(a) What is the probability we roll a three four times?

(A) 0.0156 **(B)** 0.2857 **(C)** 0.4286 **(D)** 0.5714 **(E)** 0.8988

(b) What is the probability you get at least one 5?

(A) 0.2791 **(B)** 0.6093 **(C)** 0.7209 **(D)** 0.3907 **(E)** 1

(See the solution on page 325.)

6. A student is writing a multiple-choice Statistics exam. Each question has 5 choices and only one choice is correct. There are a total of 20 questions on the exam. If the student is simply guessing on every single question:

(a) What is the probability he just barely passes the exam (gets exactly 50%)?

(A) 0.5000 **(B)** 0.1762 **(C)** 0.0026 **(D)** 0.0020 **(E)** 0.0002

(b) What is the probability he passes the exam?

(A) 0.5000 **(B)** 0.1762 **(C)** 0.0026 **(D)** 0.0020 **(E)** 0.0002

(See the solution on page 329.)

7. A seed company has determined its seeds have a 90% chance of germinating. If 20 seeds are planted what is the probability more than 18 will germinate?

(A) 0.270 **(B)** 0.285 **(C)** 0.392 **(D)** 0.608 **(E)** 0.715

(See the solution on page 331.)

8. It is known 75% of the executives at a major multinational corporation are male. In a random sample of 8 executives from this corporation, what is the probability 3 or 4 of them are female?

(A) 0.2942 **(B)** 0.0865 **(C)** 0.2076 **(D)** 0.0231 **(E)** 0.1096

(See the solution on page 333.)

9. An airline determines 97% of the people who booked a flight actually show up in time to take their seat. Assuming this is true, what is the probability, in a randomly selected sample of 12 people who independently booked various flights, no more than 10 of them showed up?

(See the solution on page 334.)

10. A newspaper reports that one in three drivers routinely exceed the speed limit. Assuming this is true, we select a random sample of 30 drivers.

(a) What is the probability exactly half of them exceed the speed limit?

(b) What is the mean number of drivers in a sample of this size who routinely exceed the speed limit, and what is the standard deviation?

(See the solution on page 336.)

11. A mail-order company finds 7% of its orders tend to be damaged in shipment. If 500 orders are shipped:

(a) Compute the mean and standard deviation of the number of orders that would be damaged.

(b) Find the approximate probability between 30 and 50 orders (inclusive) will be damaged.

(c) Find the approximate probability at least 50 orders will be damaged.

(See the solution on page 344.)

12. A recent article claimed 40% of 12-year old American children are at least 5 pounds overweight. Assuming this is true, what is the probability a random sample of 600 children finds no more than 220 12-year old American children are overweight?

(A) 0.0470 **(B)** 0.0516 **(C)** 0.9530 **(D)** 0.9484 **(E)** none of the above

(See the solution on page 348.)

13. The number of particles emitted per second by a certain radioactive substance follows a $Poisson(4)$ distribution.

(a) What is the mean and standard deviation of the number of particles emitted in a second?

(b) What is the probability two particles are emitted in a second?

(c) What is the probability less than four particles are emitted in a second?

(See the solution on page 352.)

14. The number of bacteria present in hamburger follows an approximate Poisson distribution. Suppose a large batch of hamburger has an average contamination of 0.3 bacteria/gram.

(a) The probability a 10-gram sample will contain one or fewer bacteria is:

(A) 0.2222 **(B)** 0.7408 **(C)** 0.9603 **(D)** 0.1494 **(E)** 0.1991

(b) The probability a 20-gram sample will contain exactly five bacteria is:

(A) 0.0000 **(B)** 0.1008 **(C)** 0.1606 **(D)** 0.4457 **(E)** 1.0000

(See the solution on page 354.)

15. There is an average of 1.25 spelling mistakes per page in a certain report. Assuming this follows a Poisson distribution, what is the probability there are at least 2 spelling mistakes on a randomly selected page?

(A) 0.3374 **(B)** 0.3554 **(C)** 0.6446 **(D)** 0.2865 **(E)** 0.3581

(See the solution on page 356.)

16. The number of accidents at a certain intersection follows a Poisson distribution with an average of 78 accidents per year. Assuming the number of accidents is independent of time of year, what is the probability there would be more than 2 accidents in one week?

(A) 0.0011 **(B)** 0.0405 **(C)** 0.1912 **(D)** 0.9595 **(E)** 0.8088

(See the solution on page 357.)

17. Judges on the Supreme Court of the United States of America step down at a rate of 0.3/year. Let us assume the number of judges stepping down in one year follows a Poisson distribution. President Bushton, who has the power to appoint Supreme Court justices, has just been elected for a four-year term. If he is also re-elected and completes a second term, what is the probability, during the Bushton presidency, either two, three or four judges step down?

(A) 0.1215 **(B)** 0.2613 **(C)** 0.9769 **(D)** 0.5957 **(E)** 0.3075

(See the solution on page 358.)

18. A discrete random variable X has either probability distribution p or q as shown below.

X	2	4	6	8	10
p	.02	.26	.47	.23	.02
q	.31	.01	.08	.07	.53

We are testing the hypotheses H_0: p is correct vs. H_a: q is correct. Looking at the two possible distributions, we decide to reject H_0 if $X = 2$ or 10.

(a) What is the probability of a Type I error?

(b) What is the probability of a Type II error?

(See the solution on page 362.)

19. A private home with an improperly maintained septic field can lead to contaminated water run-off which can seriously pollute rivers and lakes. An environmentalist claims at least 40% of all private septic fields near rivers are in serious need of maintenance. The city thinks this is greatly exaggerated and has hired an independent lab to select and test a random sample of 20 private homes with septic fields near the river. Let X be the number of septic fields that are in serious need of maintenance. We wish to test the hypothesis H_0: $p = 0.4$ vs. H_a: $p < 0.4$. Suppose we decide to reject H_0 if $X \leq 4$; i.e., if there are 4 or less septic fields in our sample that need maintenance.

(a) What is the Type I error in this context, and what is the probability of that error?

(b) What is the probability of a Type II error and the power of the test if the true percentage that need maintenance is 20%?

(See the solution on page 364.)

20. Let X be the number of caregivers at a hospital who call in sick on any given day. A researcher believes this follows a Poisson distribution and has set up the hypothesis H_0: $X \sim Poisson(1)$ vs. H_a: $X \sim Poisson(4)$. Suppose she decides to reject H_0 if $X \geq 3$; i.e., if 3 or more caregivers call in sick on a randomly selected day.

(a) What is the probability of a Type I error?

(b) What is the probability of a Type II error?

(See the solution on page 367.)

PREPARING FOR THE MIDTERM EXAM

- If you have done all of the homework from all 6 lessons, you are now ready to start preparing for your midterm exam. **Be sure to do <u>all</u> of the Term Tests** from the Smiley Cheng *Multiple-Choice Problems Set for Basic Statistical Analysis I (Stat 2000)* available in the Statistics section of the UM Book Store (but not the final exams obviously). **Note that the course has undergone changes in topics and philosophy over the years, so some questions in the old midterms must be omitted (you will cover those topics later in the course). Again, I will send you details of which questions are relevant to look at if you have signed up for Grant's Updates.** (I prefer to wait until the exam is approaching to make sure I know which old exam questions are relevant.) I suggest you start with the most recent exams and work your way backwards. The more recent exams are probably more indicative of what your exam will be like. The exams from the 90s are probably too easy, as the midterm has definitely gotten harder over the years.

- **The solutions to these term tests are here in Appendix C of my book starting on page C-1.**
  - Please understand many of these solutions were written several years ago, and the course has undergone many changes during that time. Specifically, you may see me referring to different statistical tables (Cheng/Fu Tables 1, 2, etc.). You are expected to use the tables in Moore & McCabe to answer all of the questions. (In the case of Poisson, of course, you have no tables, so you will have to use the Poisson probability formula.)
  - **In all of the exams prior to 2007-2008,** I use the conservative method for the two-sample t test when the pooled method is not called for. This is essentially the generalized method, but where we use the smaller of df = $n_1 - 1$ and df = $n_2 - 1$ for our degrees of freedom rather than that horrible formula we use nowadays. Back then, the course used the conservative method rather than the generalized method. You would have to use that really complicated df formula given as #1 on the formula sheet these days (but, then again, they will probably compute the df for you in that case).
  - **You may find me using the conservative *t*-test in some of the exams from the 90's even though the standard deviations suggest that you should use the pooled *t*-test** (for example, question 12 from 98/99 Term 1). This is because they were using different criteria back then. On your exam, use the "Rule of Thumb" discussed in Lesson 4 to determine if the pooled *t*-test should be used (unless your prof has told you to do otherwise).

- The **1997/98 Term 1 Midterm Exam** includes nonparametric test questions that you will not do until after the midterm, if at all. Specifically, **omit the following questions: #12 up to and including #16 <u>and</u> omit Part B #2(d)**. The **1997/98 Term 2 Midterm Exam** is almost pointless to look at. Specifically, **omit the following questions: #7 up to and including #16, #18, and #20**.

❖ **If your exam has a long answer section, be sure you do the long answer part first.** Time is sometimes an issue on the exam. If you are running out of time, you would rather be rushed as you are finishing off some multiple-choice questions (where you could always guess and hope) than feel rushed while trying to complete a more valuable long answer question. **A prepared student should have no fear of the long answer questions while there will undoubtedly be multiple-choice questions that will confuse any student.**

❖ **Never doubt yourself when answering a multiple-choice question.** If your answer is not one of the choices, simply select the closest choice and move on. Never waste your time redoing a question! If you have done it wrong, you are likely to still do it wrong the second time. You have other questions to do. Getting obsessed with one question, may mean not having time to answer two or three or more at the end. They are all worth the same marks, so leaving two or three blank at the end in order to vainly attempt to get one question right is just silly. If you have completed the exam, and still have time, by all means go back and try questions you had doubts about. Since you are now looking at the question fresh and with some distance, you have a much better chance of correcting your mistake (if you made one).

❖ **If the question is strictly theory, no math at all, you should never spend more than two minutes to make up your mind what choice to make.** Believe me. If you don't know the answer within one minute, they got you anyway, so just trust your gut, make a choice, and move on. That will buy you time to spend on the slower calculation questions.

APPENDIX A

HOW TO USE STAT MODES ON YOUR CALCULATOR

In the following pages, I show you how to enter data into your calculator in order to compute the mean and standard deviation. I also show you how to enter x, y data pairs in order to get the correlation, intercept and slope of the least squares regression line.

Please make sure that you are looking at the correct page when learning the steps. I give steps for several brands and models of calculator.

I consider it absolutely vital that a student know how to use the Stat modes on their calculator. It can considerably speed up certain questions and, even if a question insists you show all your work, gives you a quick way to check your answer.

If you cannot find steps for your calculator in this appendix, or cannot get the steps to work for you, do not hesitate to contact me. I am very happy to assist you in calculator usage (or anything else for that matter).

SHARP CALCULATORS
(Note that the EL-510 does not do Linear Regression.)

You will be using a "MODE" button. Look at your calculator. If you have "MODE" actually written on a button, press that when I tell you to press " [MODE] ". If you find mode written above a button (some models have mode written above the "DRG" button, like this: " [DRG] (MODE above) ") then you will have to use the " [2nd F] " button to access the mode button; i.e. when I say " [MODE] " below, you will actually press " [2nd F] [DRG] (MODE above) ".

BASIC DATA PROBLEM

Feed in data to get the mean, $\bar{x}$, and standard deviation, *s* (which Sharps tend to denote "*sx*").

Step 1: Put yourself into the "STAT, SD" mode.

Press [MODE] [1] [0] (Screen shows "Stat0")

Step 2: Enter the data: 3, 5, 9.
To enter each value, press the "M+" button. There are some newer models of Sharp that have you press the "CHANGE" button instead of the "M+" button. (The "CHANGE" button is found close by the "M+" button.)

3 [M+] (DATA) 5 [M+] (DATA) 9 [M+] (DATA)

You should see the screen counting the data as it is entered (Data Set=1, Data Set=2, Data Set=3).

Step 3: Ask for the mean and standard deviation.

[RCL] [4] ($\bar{x}$)

We see that $\bar{x}$ =5.6666...=5.6667.

[RCL] [5] (*sx*)

We see that *s*=3.05505...=3.0551

Step 4: Return to "NORMAL" mode. This clears out your data as well as returning your calculator to normal.

[MODE] [0]

LINEAR REGRESSION PROBLEM

Feed in *x* and *y* data to get the correlation coefficient, r, the intercept, a, and the slope, b.

Step 1: Put yourself into the "STAT, LINE" mode.

Press [MODE] [1] [1] (Screen shows "Stat1")

Step 2: Enter the data:

x	3	5	9
y	7	10	14

Note you are entering in pairs of data (the *x* and *y* must be entered as a pair). The pattern is first *x*, press "STO" to get the comma, first *y*, then press "M+" (or **"CHANGE")** to enter the pair; repeat for each data pair.

3 [STO] (x,y) 7 [M+] (DATA)

5 [STO] (x,y) 10 [M+] (DATA)

9 [STO] (x,y) 14 [M+] (DATA)

You should see the screen counting the data as it is entered (Data Set=1, Data Set=2, Data Set=3).

Step 3: Ask for the correlation coefficient, intercept, and slope. (The symbols may appear above different buttons than I indicate below.)

[RCL] [÷] (r)

We see that r=0.99419...=0.9942.

[RCL] [(] (a)

We see that a=3.85714...=3.8571.

[RCL] [)] (b)

We see that b=1.14285...=1.1429.

Step 4: Return to "NORMAL" mode. This clears out your data as well as returning your calculator to normal.

[MODE] [0]

CASIO CALCULATORS
(Note that some Casios do not do Linear Regression.)

BASIC DATA PROBLEM

Feed in data to get the mean, $\bar{x}$, and standard deviation, *s* (which Casios tend to denote "$x\sigma_{n-1}$" or simply "σ_{n-1}").

Step 1: Put yourself into the "SD" mode.

Press " MODE " once or twice until you see "SD" on the screen menu and then select the number indicated. A little "SD" should then appear on your screen.

Step 2: Clear out old data.

SHIFT AC (Scl) = (Some models will have "Scl" above another button. Be sure you are pressing "Scl", the "Stats Clear" button. (Some models call it "SAC" for "Stats All Clear" instead of Scl.)

Step 3: Enter the data: 3, 5, 9.
To enter each value, press the "M+" button.

3 M+ (DT) 5 M+ (DT) 9 M+ (DT) (You use the "M+" button to enter each piece of data.)

Step 4: Ask for the mean and standard deviation.

SHIFT 1 ($\bar{x}$) =

We see that $\bar{x}$ = 5.6666...=5.6667.

SHIFT 3 ($x\sigma_{n-1}$) =

We see that *s* = 3.05505...=3.0551

(Some models may have $\bar{x}$ and $x\sigma_{n-1}$ above other buttons rather than "1" and "3" as I illustrate above.)

If you can't find these buttons on your calculator, look for a button called "S. VAR" (which stands for "Statistical Variables", it is probably above one of the number buttons).

Press: SHIFT S. VAR and you will be given a menu showing the mean and standard deviation. Select the appropriate number on the menu and press "=" (You may need to use your arrow buttons to locate the $\bar{x}$ or $x\sigma_{n-1}$ .options.)

Step 5: Return to "COMP" mode.

Press MODE and select the "COMP" option.

LINEAR REGRESSION PROBLEM

Feed in *x* and *y* data to get the correlation coefficient, r, the intercept, a, and the slope, b.

Step 1: Put yourself into the "REG, Lin" mode.

Press " MODE " once or twice until you see "Reg" on the screen menu and then select the number indicated. You will then be sent to another menu where you will select "Lin". (Some models call it the "LR" mode in which case you simply choose that instead.)

Step 2: Clear out old data.

Do the same as Step 2 for "Basic Data".

Step 3: Enter the data.

x	3	5	9
y	7	10	14

Note you are entering in pairs of data (the *x* and *y* must be entered as a pair). The pattern is first *x*, first *y*; second *x*, second *y*; and so on. Here is the data we want to enter:

3 , 7 M+ (DT) 5 , 10 M+ (DT) 9 , 14 M+ (DT)

(If you can't find the comma button " , ", you probably use the open bracket button instead to get the comma " [(−− ". You might notice "[x_D, y_D]" in blue below this button, confirming that is your comma.)

Step 4: Ask for the correlation coefficient, intercept, and slope. (The symbols may appear above different buttons than I indicate below.)

SHIFT ((r) =

We see that r = 0.99419...=0.9942.

SHIFT 7 (A) =

We see that a = 3.85714...=3.8571.

SHIFT 8 (B) =

We see that b = 1.14285...=1.1429.

If you can't find these buttons on your calculator, look for a button called "S. VAR"

Press: SHIFT S. VAR and you will be given a menu showing the mean and standard deviation. **Use your left and right arrow buttons to see other options**, like "*r*". Select the appropriate number on the menu and press "=".

Step 5: Return to "COMP" mode.

Press MODE and select the "COMP" option.

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BASIC DATA PROBLEM

Feed in data to get the mean, $\bar{x}$, and standard deviation, *s* (which it denotes "*Sx*").

Step 1: Enter the data: 3, 5, 9.
To enter each value, press the "Σ+" button.

[3] [Σ+] [5] [Σ+] [9] [Σ+] (As you use the "Σ+" button to enter each piece of data, you will see the calculator count it going in: 1, 2, 3.)

Step 2: Ask for the mean and standard deviation.
Note that by "orange" I mean press the button that has the orange bar coloured on it. The orange bar is used to get anything coloured orange on the buttons.

[orange] [7 $\bar{x},\bar{y}$]

We see that $\bar{x}$ = 5.6666...=5.6667.

[orange] [8 s_x, s_y]

We see that *s* = 3.05505...=3.0551

Step 3: "Clear All" data ready for next time.

[orange] [C C ALL]

LINEAR REGRESSION PROBLEM

Feed in *x* and *y* data to get the correlation coefficient, r, the intercept, a, and the slope, b.

Step 1: Enter the data:

x	3	5	9
y	7	10	14

Note you are entering in pairs of data (the *x* and *y* must be entered as a pair). The pattern is first *x*, first *y*; second *x*, second *y*; and so on.

[3] [INPUT] [7] [Σ+]

[5] [INPUT] [10] [Σ+]

[9] [INPUT] [14] [Σ+]

(As you use the "Σ+" button to enter each pair of data, you will see the calculator count it going in: 1, 2, 3.)

Step 2: Ask for the correlation coefficient, intercept, and slope.

[orange] [4 $\hat{x},r$] [orange] [K SWAP]

We see that *r* = 0.99419...=0.9942.
Note that the "SWAP" button is used to get anything that is listed second (after the comma) like "*r*" in this case.

The intercept has to be found by finding $\hat{y}$ when *x*=0:

0 [orange] [5 $\hat{y},m$]

We see that *a* = 3.85714...=3.8571.

The slope is denoted "*m*" on this calculator:

[orange] [5 $\hat{y},m$] [orange] [K SWAP]

We see that *b* = 1.14285...=1.1429.

Step 3: "Clear All" data ready for next time.

[orange] [C C ALL]

TEXAS INSTRUMENTS TI-30X-II
(Note that the TI-30Xa does not do Linear Regression.)

BASIC DATA PROBLEM

Feed in data to get the mean, $\bar{x}$, and standard deviation, s (which it denotes "*Sx*").

Step 1: Clear old data.

[2nd] [DATA] (STAT) Use your arrow keys to ensure "*CLRDATA*" is underlined then press [ENTER =]

Step 2: Put yourself into the "STAT 1-Var" mode.

[2nd] [DATA] (STAT) Use your arrow keys to ensure "1-Var" is underlined then press [ENTER =]

Step 3: Enter the data: 3, 5, 9.

(You will enter the first piece of data as "X1", then use the down arrows to enter the second piece of data as "X2", and so on.)

[DATA] 3 [ENTER =] (X1 = 3)

[⇩] [⇩] 5 [ENTER =] (X2 = 5)

[⇩] [⇩] 9 [ENTER =] (X3 = 9)

Step 4: Ask for the mean and standard deviation.

Press [STATVAR] then you can see a list of outputs by merely pressing your left and right arrows to underline the various values.

We see that $\bar{x}$ = 5.6666...=5.6667.

We see that s = 3.05505...=3.0551

Step 5: Return to standard mode.

[CLEAR] This resets your calculator ready for new data next time.

LINEAR REGRESSION PROBLEM

Feed in *x* and *y* data to get the correlation coefficient, r, the intercept, a, and the slope, b.

Step 1: Clear old data (as in **BASIC DATA PROBLEM** at left).

Step 2: Put yourself into the "STAT 2-Var" mode.

[2nd] [DATA] (STAT) Use your arrow keys to ensure "2-Var" is underlined then press [ENTER =]

Step 3: Enter the data:

x	3	5	9
y	7	10	14

(You will enter the first *x*-value as "X1", then use the down arrow to enter the first *y*-value as "Y1", and so on.)

[DATA] 3 [ENTER =] [⇩] 7 [ENTER =] (X1 = 3, Y1 = 7)

[⇩] 5 [ENTER =] [⇩] 10 [ENTER =] (X2 = 5, Y2 = 10)

[⇩] 9 [ENTER =] [⇩] 14 [ENTER =] (X3 = 9, Y3 = 14)

Step 4: Ask for the correlation coefficient, intercept, and slope.

Press [STATVAR] then you can see a list of outputs by merely pressing your left and right arrows to underline the various values. **Note: Your calculator may have a and b reversed. To get a, you ask for b; to get b you ask for a.** Don't ask me why that is, but if that is the case then realize it will <u>always</u> be the case.

We see that r = 0.99419...=0.9942.

We see that a = 3.85714...=3.8571.

We see that b = 1.14285...=1.1429.

Step 5: Return to standard mode (as in **BASIC DATA PROBLEM** at left).

TEXAS INSTRUMENTS TI-36X
(Note that the TI-30Xa does not do Linear Regression.)

BASIC DATA PROBLEM

Feed in data to get the mean, $\bar{x}$, and standard deviation, *s* (which it denotes "σx_{n-1}").

Step 1: Put yourself into the "STAT 1" mode.

[3rd] [x ⇄ y] (STAT 1)

Step 2: Enter the data: 3, 5, 9.
To enter each value, press the "Σ+" button.

3 [Σ+] 5 [Σ+] 9 [Σ+] (As you use the "Σ+" button to enter each piece of data, you will see the calculator count it going in: 1, 2, 3.)

Step 3: Ask for the mean and standard deviation.

[2nd] [x^2] ($\bar{x}$)

We see that $\bar{x}$ = 5.6666...=5.6667.

[2nd] [$\sqrt{x}$] (σ_{xn-1})

We see that *s* = 3.05505...=3.0551

Step 4: Return to standard mode.

[ON/AC] (Be careful! If you ever press this button during your work you will end up resetting your calculator and losing all of your data. Use the [CE/C] button to clear mistakes without resetting your calculator. I usually press this button a couple of times to make sure it has cleared any mistake completely.)

LINEAR REGRESSION PROBLEM

Feed in *x* and *y* data to get the correlation coefficient, r, the intercept, a, and the slope, b.

Step 1: Put yourself into the "STAT 2" mode.

[3rd] [Σ+] (STAT 2)

Step 2: Enter the data:

x	3	5	9
y	7	10	14

Note you are entering in pairs of data (the *x* and *y* must be entered as a pair). The pattern is first *x*, first *y*; second *x*, second *y*; and so on.

3 [$x \rightleftarrows y$] 7 [Σ+]

5 [$x \rightleftarrows y$] 10 [Σ+]

9 [$x \rightleftarrows y$] 14 [Σ+]

(As you use the "Σ+" button to enter each pair of data, you will see the calculator count it going in: 1, 2, 3.)

Step 3: Ask for the correlation coefficient, intercept, and slope.

Note that this calculator uses the abbreviations "COR" for correlation, "ITC" for intercept and "SLP" for slope.

[3rd] [4] (COR)

We see that r = 0.99419...=0.9942.

[2nd] [4] (ITC)

We see that a = 3.85714...=3.8571.

[2nd] [5] (SLP)

We see that b = 1.14285...=1.1429.

Step 4: Return to standard mode.

[ON/AC]

TEXAS INSTRUMENTS TI-BA II Plus

Put yourself into the "LIN" mode.

[2nd] [8] (STAT) If "LIN" appears, great; if not, press [2nd] [ENTER] (SET) repeatedly until "LIN" does show up. Then press [2nd] [CPT] (QUIT) to "quit" this screen.

Note: Once you have set the calculator up in "LIN" mode, it will stay in that mode forever. You can now do either "Basic Data" or "Linear Regression" problems.

BASIC DATA PROBLEM

Feed in data to get the mean, $\bar{x}$, and standard deviation, *s* (which it denotes "*Sx*").

Step 1: Clear old data.

[2nd] [7] (DATA) [2nd] [CE/C] (CLR Work)

Step 2: Enter the data: 3, 5, 9.

(You will enter the first piece of data as "X1", then use the down arrows to enter the second piece of data as "X2", and so on. Ignore the "Y1", "Y2", etc.)

[DATA] 3 [ENTER =] $(X1=3)$

[⇩] [⇩] 5 [ENTER =] $(X2=5)$

[⇩] [⇩] 9 [ENTER =] $(X3=9)$

Step 3: Ask for the mean and standard deviation.

Press [2nd] [8] (STAT) then you can see a list of outputs by merely pressing your up and down arrows to reveal the various values.

We see that $\bar{x}$ = 5.6666...=5.6667.

We see that *s* = 3.05505...=3.0551

Step 4: Return to standard mode.

[ON/OFF] This resets your calculator ready for new data next time.

LINEAR REGRESSION PROBLEM

Feed in *x* and *y* data to get the correlation coefficient, r, the intercept, a, and the slope, b.

Step 1: Clear old data.

[2nd] [7] (DATA) [2nd] [CE/C] (CLR Work)

Step 2: Enter the data:

x	3	5	9
y	7	10	14

(You will enter the first *x*-value as "X1", then use the down arrow to enter the first *y*-value as "Y1", and so on.)

[DATA] 3 [ENTER =] [⇩] 7 [ENTER =] $(X1=3, Y1=7)$

[⇩] 5 [ENTER =] [⇩] 10 [ENTER =] $(X2=5, Y2=10)$

[⇩] 9 [ENTER =] [⇩] 14 [ENTER =] $(X3=9, Y3=14)$

Step 3: Ask for the correlation coefficient, intercept, and slope.

Press [2nd] [8] (STAT) then you can see a list of outputs by merely pressing your up and down arrows to reveal the various values.

We see that r = 0.99419...=0.9942.

We see that a = 3.85714...=3.8571.

We see that b = 1.14285...=1.1429.

Step 4: Return to standard mode.

[ON/OFF] This resets your calculator ready for new data next time.