Grant's Tutoring

BASIC STATISTICS 2

Volume 1 of 2

September 2014 edition

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the entire Basic Statistics 2 course
has been split into two volumes.

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HOW TO USE THIS BOOK

I have broken the course up into lessons. Do note that the numbering of my lessons do not necessarily correspond to the numbering of the units in your course outline. Study each lesson until you can do all of my lecture problems from start to finish without any help. If you are able to solve my Lecture Problems, then you should have nothing to fear about your exams.

Although NOT ESSENTIAL, you may want to purchase the *Multiple-Choice Problems Set for Basic Statistical Analysis II (Stat 2000)* by Dr. Smiley Cheng. This book is now out of print, but copies may be available at The Book Store. The appendices of my book include complete step-by-step solutions for all the problems and exams in Cheng's book. **Be sure to read the "Homework" section at the end of each lesson for important guidance from me on how to proceed in your studying, as there has been changes to the course since the Cheng book was published.**

You also need a good, but not expensive, scientific calculator. Any of the makes and models of calculators I discuss in Appendix A are adequate for this course. **Appendix A in this book shows you how to use all major models of calculators.**

I have presented the course in what I consider to be the most logical order. Although my books are designed to follow the course syllabus, it is possible your prof will teach the course in a different order or omit a topic. It is also possible he/she will introduce a topic I do not cover. **Make sure you are attending your class regularly! Stay current with the material, and be aware of what topics are on your exam. Never forget, it is your prof that decides what will be on the exam, so pay attention.**

If you have any questions or difficulties while studying this book, or if you believe you have found a mistake, do not hesitate to contact me. My phone number and website are noted at the bottom of every page in this book. "Grant's Tutoring" is also in the phone book. **I welcome your input and questions.**

Wishing you much success,

Grant Skene

Owner of Grant's Tutoring and author of this book

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IT IS POSSIBLE THAT YOUR MIDTERM EXAM MAY ALSO INCLUDE SOME OF THE CONTENT IN VOLUME 2 OF MY BOOK.

Appendices

FORMULA SHEET

A formula sheet is included in your exams. Check your course syllabus and compare it to the formula sheet I use below in case the formula sheet in your course has changed.

1. $SE\left(\bar{x}_1-\bar{x}_2\right)=\sqrt{\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}}$ with $\mathrm{df}=\dfrac{\left(\frac{s_1^2}{n_1}+\frac{s_2^2}{n_2}\right)^2}{\frac{1}{n_1-1}\left(\frac{s_1^2}{n_1}\right)^2+\frac{1}{n_2-1}\left(\frac{s_2^2}{n_2}\right)^2}$

2. $SE\left(\bar{x}_1-\bar{x}_2\right)=s_p\sqrt{\frac{1}{n_1}+\frac{1}{n_2}}$ with $\mathrm{df}=n_1+n_2-2$ if $\sigma_1^2=\sigma_2^2$

where $s_p^2=\dfrac{(n_1-1)s_1^2+(n_2-1)s_2^2}{n_1+n_2-2}$

3. $SSG=\sum_{i=1}^{k}n_i\left(\overline{X}_i-\overline{\overline{X}}\right)^2$

4. Poisson Distribution: $P\left(X=k\right)=\dfrac{e^{-\lambda}\lambda^k}{k!}$ $\quad k=0, 1, 2, \ldots$

5. $t=\dfrac{r\sqrt{n-2}}{\sqrt{1-r^2}}$

6. $SE_{b_1}=\dfrac{s_e}{\sqrt{\sum\left(x_i-\bar{x}\right)^2}}$, $s_e=\sqrt{MSE}$

7. $SE_{b_0}=s_e\sqrt{\frac{1}{n}+\frac{\bar{x}^2}{\sum\left(x_i-\bar{x}\right)^2}}$

8. $SE_{\hat{\mu}}=s_e\sqrt{\frac{1}{n}+\frac{\left(x^*-\bar{x}\right)^2}{\sum\left(x_i-\bar{x}\right)^2}}$

9. $SE_{\hat{y}}=s_e\sqrt{1+\frac{1}{n}+\frac{\left(x^*-\bar{x}\right)^2}{\sum\left(x_i-\bar{x}\right)^2}}$

10. $SE\left(\hat{p}_1-\hat{p}_2\right)=\sqrt{\hat{p}\left(1-\hat{p}\right)\left(\frac{1}{n_1}+\frac{1}{n_2}\right)}$ if $p_1=p_2$ where $\hat{p}=\dfrac{x_1+x_2}{n_1+n_2}$

11. $SE\left(\hat{p}_1-\hat{p}_2\right)=\sqrt{\frac{\hat{p}_1\left(1-\hat{p}_1\right)}{n_1}+\frac{\hat{p}_2\left(1-\hat{p}_2\right)}{n_2}}$ if $p_1\neq p_2$

STEPS FOR TESTING A HYPOTHESIS

Step 1. State the null and alternative hypotheses (H_0 and H_a), and so determine if the test is 2-tailed, upper-tailed, or lower-tailed.

Step 2. Use the given α (always use α = 5% if none is given) to get the **critical value** (z^*, t^*, F^*, etc. depending on the hypothesis you are testing) from the appropriate table and state the **rejection region**.

Step 3. Compute the **test statistic** (z, t, F, etc. depending on the hypothesis you are testing) using the appropriate formula, and see if it lies in the rejection region.

Step 4. (Only if specifically asked to do so.) Compute the ***P*-value**.

Draw a density curve (z-bell curve, t-bell curve, F right-skewed curve, etc. depending on the test statistic you have computed), mark the test statistic (found in Step 3), and shade the area as instructed by H_a. That area is the P-value.

Remember, a P-value is very handy to know if you are asked to make decisions for more than one value of α.

Reject H_0 if P-value < α.

Step 5. State your conclusion.

Either: Reject H_0. There is statistically significant evidence that the alternative hypothesis is correct. (Replace the underlined part with appropriate wording from the problem that says H_a is correct.)

Or: Do not reject H_0. There is no statistically significant evidence that the alternative hypothesis is correct. (Replace the underlined part with appropriate wording from the problem that says we are not convinced that H_a is correct.)

SUMMARY OF KEY FORMULAS IN THIS COURSE

Lesson 1. The mean and standard deviation of $\bar{x}$ are $\mu_{\bar{x}} = \mu$ and $\sigma_{\bar{x}} = \dfrac{\sigma}{\sqrt{n}}$.

The standard error of $\bar{x} = SE_{\bar{x}} = \dfrac{s}{\sqrt{n}}$.

Central Limit Theorem: If n is large, $\bar{x}$ is approximately normal.

Confidence Intervals for μ: $\bar{x} \pm z^* \dfrac{\sigma}{\sqrt{n}}$ or $\bar{x} \pm t^* \dfrac{s}{\sqrt{n}}$

Sample size determination: $n = \left(\dfrac{z^* \sigma}{m}\right)^2$

Lesson 2. Test statistics for H_0: $\mu = \mu_0$ are $z = \dfrac{\bar{x} - \mu_0}{\sigma/\sqrt{n}}$ or $t = \dfrac{\bar{x} - \mu_0}{s/\sqrt{n}}$.

Lesson 3. Standardizing formula for $\bar{x}$ bell curves: $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$.

To compute $\bar{x}^*$ for $\bar{x}$ decision rules: $\bar{x}^* = z^* \dfrac{\sigma}{\sqrt{n}} + \mu_0$.

Lesson 4. Properties for means of two random variables:

$$\mu_{X+Y} = \mu_X + \mu_Y$$
$$\mu_{X-Y} = \mu_X - \mu_Y$$

Properties for variance of two independent random variables:

$$\sigma^2_{X+Y} = \sigma^2_X + \sigma^2_Y$$
$$\sigma^2_{X-Y} = \sigma^2_X + \sigma^2_Y$$

Properties for variance of two dependent random variables with correlation ρ:

$$\sigma^2_{X+Y} = \sigma^2_X + \sigma^2_Y + 2\rho\sigma_X\sigma_Y$$
$$\sigma^2_{X-Y} = \sigma^2_X + \sigma^2_Y - 2\rho\sigma_X\sigma_Y$$

Confidence interval for $\mu_1 - \mu_2$ is $(\bar{x}_1 - \bar{x}_2) \pm t^* SE(\bar{x}_1 - \bar{x}_2)$.

To test H_0: $\mu_1 = \mu_2$, the test statistic is $t = \dfrac{\bar{x}_1 - \bar{x}_2}{SE(\bar{x}_1 - \bar{x}_2)}$.

The formulas for the degrees of freedom and $SE(\bar{x}_1 - \bar{x}_2)$ are included on the Formula Sheet given on your exams (page 1 of this book).

SUMMARY OF KEY FORMULAS IN THIS COURSE (CONTINUED)

Lesson 5. $DFG = I - 1$ and $DFE = N - I$

$$\bar{\bar{x}} = \frac{\sum n_i \bar{x}_i}{N} = \frac{n_1\bar{x}_1 + n_2\bar{x}_2 + n_3\bar{x}_3 + \ldots + n_I\bar{x}_I}{N}$$

$MSG = \dfrac{SSG}{DFG}$ (The formula for SSG is included on the Formula Sheet given on your exams (page 1 of this book).)

$$MSE = \frac{SSE}{DFE} = \frac{\sum (n_i - 1)s_i^2}{N - I} = \frac{(n_1 - 1)s_1^2 + (n_2 - 1)s_2^2 + \ldots + (n_I - 1)s_I^2}{N - I}$$

$s_p^2 = MSE$. (The Formula Sheet given on your exams (page 1 of this book) gives you the two-sample version of the s_p^2 formula; this formula can be generalized for three, four or more samples.)

The formula for the F test statistic is $F = \dfrac{MSG}{MSE}$ with df = DFG, DFE.

The coefficient of determination = $R^2 = \dfrac{SSG}{SST}$.

Lesson 6. If X is a discrete random variable:

$$\mu = \sum x\,p(x) \text{ and } \sigma^2 = \sum (x - \mu)^2 p(x) \text{ or } \sigma^2 = \left(\sum x^2 p(x)\right) - \mu^2$$

<u>The Complement Rule</u>: $P(A^C) = 1 - P(A)$

<u>The General Addition Rule</u>: $P(A \text{ or } B) = P(A) + P(B) - P(A \text{ and } B)$

<u>"Neither/nor" is 1 minus "or"</u>: $P(\text{neither } A \text{ nor } B) = 1 - P(A \text{ or } B)$

<u>The General Multiplication Rule</u>: $P(A \text{ and } B) = P(A) \times P(B|A)$

<u>If A and B are disjoint</u>, then $P(A \text{ and } B) = 0$.

<u>If (and only if) A and B are independent</u>, then $P(A \text{ and } B) = P(A) \times P(B)$.

<u>Conditional Probability</u>: $$P(B|A) = \frac{P(B \text{ and } A)}{P(A)}$$

LESSON 3: ERRORS IN HYPOTHESIS TESTING

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TYPE I AND TYPE II ERRORS

Every time we test a hypothesis, we never know what the true situation is. All we can do is take a sample and, using our steps for testing hypotheses, decide whether to reject H_0 or not. Even though we have done everything properly, there is always a chance we have made an incorrect decision. We have not done anything wrong; it is just that our sample misled us. There are two types of errors we might make in testing a hypothesis.

Memorize these definitions:

TYPE I ERROR: Rejecting H_0 when H_0 is true.

TYPE II ERROR: Not rejecting H_0 when H_0 is false.

The *probability* of a Type I error is denoted "α". This is the *same* α that denotes the level of significance. The researcher chooses α's value for a hypothesis test, and so has complete control over the probability of making a Type I error. If people could get hurt or killed by an incorrect decision, statisticians always make H_0 the *safe* hypothesis. Which is to say, if they stick with H_0, even incorrectly, at least nobody dies. They then choose a very *small* value of α (say $\alpha = 1\%$ or even $\alpha = .01\%$) so that there is a *low* probability of a Type I error.

For example, let's say we are testing someone for cancer. We are most worried about telling someone they don't have cancer when, in fact, they do. Obviously, that mistake could seriously jeopardize that person's health, as well as endangering others. Therefore, we set up the following hypotheses:

H_0: The person has cancer.

H_a: The person does not have cancer.

A Type I error (rejecting H_0 when H_0 is true) in this context would be devastating. That means we are concluding a person does not have cancer, when in fact they do. Which is to say, our test has given us a **false negative**. Now this person's cancer is going untreated, perhaps resulting in their death.

It is impossible to prevent this mistake, but we can lessen the chance it happens. If we make α = .01% = .0001, for example, this false negative would occur only .01% of the time we test someone with cancer; P(Type I error) = .01%.

Of course, the standard value is α = 5% in hypothesis testing which is, relatively speaking, a reasonable risk of Type I error for ordinary statistical experiments. Usually, lives aren't at stake when we are testing a hypothesis, so a 5% chance of Type I error is tolerable.

The *probability* of Type II error is denoted β (beta, the Greek letter "b"). **We have no direct control over β's value, we can only affect β indirectly. Knowing the value of α, the probability of Type I error, in no way tells you β, the probability of Type II error.** However, there is this relationship:

If we increase α, β decreases.
If we decrease α, β increases.

For example, in the cancer example above, we decreased α from its standard value of 5% to a mere .01%. However, this has caused β to increase. By lowering the probability of a Type I error, we have inevitably increased the probability of a Type II error. Recall, a Type II error is not rejecting H_0 when H_0 is false. In this context, that means concluding someone has cancer, when in fact they do not. Here, by lowering the probability of a false negative, we have increased the probability of a **false positive**.

Obviously, this mistake would cause someone unnecessary anxiety, but we would counsel them immediately, telling them our test has many false positives. We would just tell them to take precautions for the next little while until we complete more comprehensive tests to see if the person does, in fact, have cancer.

Medical tests are always set up to give few false negatives at the expense of more false positives. Better that some healthy people have a few sleepless nights after a preliminary test shows they may have cancer than someone who has cancer going undetected. Sadly, we can never reduce false negatives to zero. The only way to do that is to immediately diagnose everyone with cancer the minute they step in the door.

If a researcher believes one error would have much more serious consequences than the other, they should set up their null and alternative hypotheses in such a way that the Type I error would be the more serious one. We have much more direct control over the probability of a Type I error since it equals α, the level of significance that we choose ourselves.

THE POWER OF A TEST

The power of a test is the *probability* we will *correctly* reject H_0.

When we are testing a hypothesis, if H_0 is wrong, we want our research to be able to convince us of that. Which is to say, we want a *powerful* test; a test with a high power.

The probability of Type II error, β, and power are related.

Power = 1 – β

Which is to say, β and power add up to 100%. Do not take this formula too literally. This does not mean you have to compute β first, and then go "1 – β" to find the power. You could just as easily find power first, then go "1 – power" to get β. The key thing to understand is, once you know one of these guys, you know the other.

A reliable hypothesis test will have low probabilities of error (low α and low β) and a high probability of correctly rejecting H_0 (high power). This is what a researcher strives for. The best way to accomplish this is to use a large sample size (large n). Large sample sizes give you more trustworthy results, decreasing the chance of any error, and increasing the chance of correct decisions.

Essentially, a researcher has control over two things. He/she can decide what the level of significance, α, will be (thus controlling the probability of Type I error), and he/she can decide on how large a sample size, n to use. **Altering the values of α or n set up a "chain reaction".**

The "α/β/Power" Chain

If $\alpha\downarrow$, then $\beta\uparrow$, power $\downarrow$.
If $\alpha\uparrow$, then $\beta\downarrow$, power $\uparrow$.

If we <u>de</u>crease α, that <u>in</u>creases β, the probability of Type II error, which then <u>de</u>creases the power of the test. Similarly, if we <u>in</u>crease α, that <u>de</u>creases β, which then <u>in</u>creases the power of the test.

The "n/β/Power" Chain

If $n\uparrow$, then $\beta\downarrow$, power $\uparrow$.
If $n\downarrow$, then $\beta\uparrow$, power $\downarrow$.

If we <u>in</u>crease n, that <u>de</u>creases β, which then <u>in</u>creases the power of the test. Similarly, if we <u>de</u>crease n, that <u>in</u>creases β, which then <u>de</u>creases the power of the test.

Let's try some questions.

1. For the table below, indicate which are correct decisions and which are errors. If they are errors, what type?

		True Situation	
		H_0 is true	H_0 is false
Decision	Reject H_0		
	Not to reject H_0		

Profs often like to make a chart like this to illustrate the errors and correct decisions for hypothesis tests. If you find yourself needing to fill in something like this, I recommend you first find the cell which represents a Type I error (rejecting H_0 when H_0 is true). Depending on how they have labelled the columns and rows of the table, any cell might represent this error. In this particular problem, the cell in the top left corner says it is rejecting H_0 even though H_0 is true, so that is the Type I error.

Once you have found the Type I error cell, it is guaranteed the Type II error cell will always be diagonally across from it (no matter how the columns and rows have been labelled). The remaining two cells will always represent correct decisions. Fill in the table.

Solution to Question 1

		True Situation	
		H_0 is true	H_0 is false
Decision	Reject H_0	Type I error	Correct decision
	Not to reject H_0	Correct decision	Type II error

2. In testing a hypothesis you decide to lower the level of significance from 5% to 1%. Consider the following statements:

(I) The probability of Type I error has decreased from .05 to .01.

(II) The probability of Type II error has increased from .95 to .99.

(III) The power of the test has increased from .95 to .99.

(IV) The probability of Type II error has increased, but the amount cannot be determined.

(V) The probability of Type I error has decreased, but the amount cannot be determined.

(A) only (I) is true
(B) only (I) and (II) are true
(C) only (I) and (III) are true
(D) only (I) and (IV) are true
(E) only (II) and (V) are true

(I) is **TRUE**. We are told α has been lowered from 5% to 1%, so, since α is the probability of Type I error, we know *P*(Type I error) has been lowered from 5% to 1%.

(II) is **FALSE**. We know α has <u>decreased</u>, so β, the probability of Type II error, has <u>increased</u>, but we have absolutely no idea what β's value might be. **Knowing the value of α in no way tells you the value of β.**

(III) is **FALSE**. We know **power = 1 –** β, but since we have no idea what value β has, we also could not possibly know the power. In addition, we know:

If $\alpha\downarrow$, then $\beta\uparrow$, power $\downarrow$.

So, the power will decrease not increase.

(IV) is **TRUE**. Again, **if $\alpha\downarrow$, then $\beta\uparrow$, power $\downarrow$.**

(V) is **FALSE**. We know α's value exactly.

Solution to Question 2

(I) and (IV) are true. The correct answer is (D).

THE $\bar{x}$ DECISION RULE

If we want to compute P(Type I error), P(Type II error) or the power of a hypothesis test for μ, first and foremost we must know the **$\bar{x}$ decision rule**.

Decision rules tell us when to reject H_0 (and, by omission, also tell us when not to reject H_0). So far, we have seen decision rules based on z ("Reject H_0 if $z > 1.645$" is an example of a "**z decision rule**"); decision rules based on t ("Reject H_0 if $t < -1.711$" is an example of a "**t decision rule**"); we also have used decision rules based on the P-value ("Reject H_0 if the P-value < 5%" is an example of a "***P*-value decision rule**").

An example of an "$\bar{x}$ decision rule" for an upper-tailed test would be something like "Reject H_0 if $\bar{x} > 12.6$". Or, for a lower-tailed test, you might have something like "Reject H_0 if $\bar{x} < 6.8$".

Of course, to state a z or t decision rule, we need to use Table D* to get the z or t critical value, z^* or t^*, first. Similarly, to state an $\bar{x}$ decision rule, we need to know the $\bar{x}$ critical value, $\bar{x}^*$, first. In the example "Reject H_0 if $\bar{x} > 12.6$", the critical value is 12.6 (**$\bar{x}^* = 12.6$**); whereas "Reject H_0 if $\bar{x} < 6.8$" tells us the critical value is 6.8 (**$\bar{x}^* = 6.8$**).

We can't simply get $\bar{x}^*$ off a table. We have to use a little algebra. You can take it for granted all "α, β, power" problems involving the mean, μ, will use z instead of t (which is to say, you will be given σ in the problems). This is because only z can enable us to compute *exact* probabilities. If we are using t, the best we can do is put bounds on the probability (like we do for P-values). The profs will want us to calculate exact probabilities, and so will always set up the problem to allow the use of z.

Take the z test statistic formula and isolate $\bar{x}$.

$$z = \frac{\bar{x} - \mu_0}{\sigma / \sqrt{n}}$$

Multiply the "$\frac{\sigma}{\sqrt{n}}$" in the denominator diagonally across to the other side.

* I am referring to "Table D: t distribution critical values" in Moore & McCabe's *Introduction to the Practice of Statistics*. This table will be used throughout this book to find z^* and t^* critical values.

$$z\frac{\sigma}{\sqrt{n}} = \bar{x} - \mu_0$$

Move the "– μ_0" over to the other side to isolate $\bar{x}$.

$$z\frac{\sigma}{\sqrt{n}} + \mu_0 = \bar{x} \qquad \text{or} \qquad \bar{x} = z\frac{\sigma}{\sqrt{n}} + \mu_0$$

But, we specifically want to know, the critical value, $\bar{x}^*$, so we use the critical z value, z^* to get it. Simply stick a "*" next to the $\bar{x}$ and z in the formula we have derived.

The formula to compute $\bar{x}^*$ is $\quad \bar{x}^* = z^*\frac{\sigma}{\sqrt{n}} + \mu_0$

(I strongly recommend you memorize this formula.)

TO STATE THE "$\bar{x}$ DECISION RULE":

Step 1. **Use Table D to get z^*** for the given α. (Remember: z^* is <u>negative</u> if you have a lower-tailed rejection region!)

Step 2. **Now compute $\bar{x}^*$.** $\quad \bar{x}^* = z^*\frac{\sigma}{\sqrt{n}} + \mu_0$

Step 3. **State the $\bar{x}$ decision rule.**

If the test is upper-tailed: **Reject H_0 if $\bar{x} > \bar{x}^*$**

If the test is lower-tailed: **Reject H_0 if $\bar{x} < \bar{x}^*$**

If the test is two-tailed, you will have to compute two different values of $\bar{x}^*$, a lower-tailed value ($\bar{x}_L^*$) using the NEGATIVE z^* critical value, and an upper-tailed value ($\bar{x}_U^*$) using the POSITIVE z^* critical value.

If the test is two-tailed: **Reject H_0 if $\bar{x} < \bar{x}_L^*$ or $\bar{x} > \bar{x}_U^*$**

3. A researcher believes the average female executive at any company with more than 500 employees is making less than the 100 thousand dollars a year the average male executive makes. Consequently, she is going to test the hypotheses H_0: μ = 100 thousand dollars vs. H_a: μ < 100 thousand dollars. From previous research, it is known the distribution of executive salaries is approximately normal with σ = 20 thousand dollars. Determine the $\bar{x}$ decision rule in the following cases:

(a) She will select a random sample of 16 female executives and use a 10% level of significance.

(b) She will select a random sample of 16 female executives and use a 5% level of significance.

(c) She will select a random sample of 64 female executives and use a 5% level of significance.

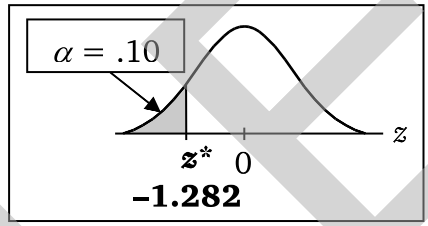
(d) She will select a random sample of 64 female executives and use a 2% level of significance.

Givens: $\boldsymbol{H_0}$: μ = **100** (we are given μ_0 = **100**) vs. $\boldsymbol{H_a}$: μ < **100** (a **lower-tailed** test); **normal distribution** (we can draw an $\bar{x}$-bell curve); $\boldsymbol{\sigma}$ **= 20** (we can use z). We are asked to determine the $\bar{x}$ decision rule in each case, so we will first need to compute $\bar{x}^*$, the critical $\bar{x}$ value using the formula $\bar{x}^* = z^* \dfrac{\sigma}{\sqrt{n}} + \mu_0$.

3. (a) She will select a random sample of 16 female executives and use a 10% level of significance.

<u>**Givens**</u>**:** $\boldsymbol{n}$ **= 16**; $\boldsymbol{\alpha}$ **= 10%**. We already know $\boldsymbol{\mu_0}$ **= 100** and $\boldsymbol{\sigma}$ **= 20**.

First we need to know z^*, the critical z value. We get z^* the way we usually do in hypothesis testing (as we learned in Lesson 2). We are conducting a **lower-tailed test at α = 10%**, so visualize 10% = .10 shaded in the lower tail of the z-bell curve. Consulting Table D at the ".10" column, we see the UPPER-TAILED critical value is z^* = 1.282, but **WE ARE DOING A LOWER-TAILED TEST! THE z^* CRITICAL VALUE IS NEGATIVE! z^* = –1.282**.



$$\bar{x}^* = z^* \frac{\sigma}{\sqrt{n}} + \mu_0 = -1.282 \times \frac{20}{\sqrt{16}} + 100 = \mathbf{93.59}$$

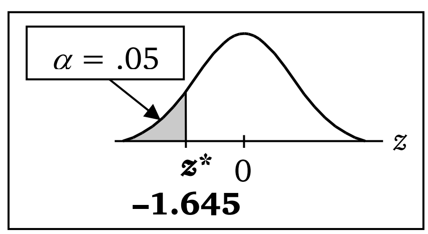
Solution to Question 3 (a)

The $\bar{x}$ decision rule is "Reject H_0 if $\bar{x}$ < 93.59."

3. (b) She will select a random sample of 16 female executives and use a 5% level of significance.

<u>**Givens**</u>**:** $\boldsymbol{n}$ **= 16**; $\boldsymbol{\alpha}$ **= 5%**. We already know $\boldsymbol{\mu_0}$ **= 100** and $\boldsymbol{\sigma}$ **= 20**.

First we need to know z^*, the critical z value. We are conducting a **lower-tailed test at α = 5%**, so visualize 5% = .05 shaded in the lower tail of the z-bell curve. Consulting Table D at the ".05" column, we see the UPPER-TAILED critical value is z^* = 1.645, but **WE ARE DOING A LOWER-TAILED TEST! THE z^* CRITICAL VALUE IS NEGATIVE! z^* = –1.645**.



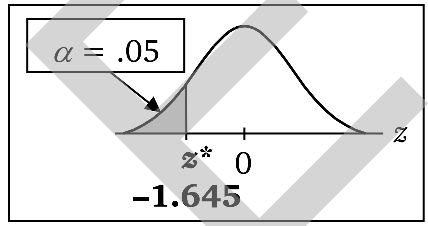
$$\bar{x}^* = z^* \frac{\sigma}{\sqrt{n}} + \mu_0 = -1.645 \times \frac{20}{\sqrt{16}} + 100 = \mathbf{91.775}$$

Solution to Question 3 (b)

The $\bar{x}$ decision rule is "Reject H_0 if $\bar{x}$ < 91.775."

3. (c) She will select a random sample of 64 female executives and use a 5% level of significance.

<u>**Givens**</u>**:** $\boldsymbol{n}$ **= 64**; $\boldsymbol{\alpha}$ **= 5%**. We already know $\boldsymbol{\mu_0}$ **= 100** and $\boldsymbol{\sigma}$ **= 20**.



First we need to know z^*, the critical z value. We are conducting a **lower-tailed test at α = 5%**, so visualize 5% = .05 shaded in the lower tail of the z-bell curve. Consulting Table D at the ".05" column, we see the UPPER-TAILED critical value is z^* = 1.645, but **WE ARE DOING A LOWER-TAILED TEST! THE z^* CRITICAL VALUE IS NEGATIVE! z^* = –1.645**.

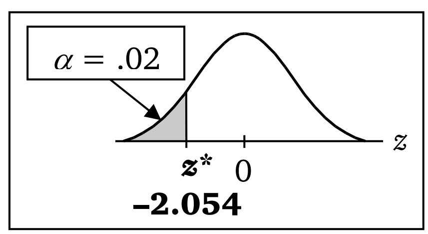
$$\bar{x}^* = z^* \frac{\sigma}{\sqrt{n}} + \mu_0 = -1.645 \times \frac{20}{\sqrt{64}} + 100 = \mathbf{95.8875}$$

Solution to Question 3 (c)

The $\bar{x}$ decision rule is "Reject H_0 if $\bar{x}$ < 95.8875."

3. (d) She will select a random sample of 64 female executives and use a 2% level of significance.

<u>**Givens**</u>**:** $\boldsymbol{n}$ **= 64**; $\boldsymbol{\alpha}$ **= 2%**. We already know $\boldsymbol{\mu_0}$ **= 100** and $\boldsymbol{\sigma}$ **= 20**.



First we need to know z^*, the critical z value. We are conducting a **lower-tailed test at α = 2%**, so visualize 2% = .02 shaded in the lower tail of the z-bell curve. Consulting Table D at the ".02" column, we see the UPPER-TAILED critical value is z^* = 2.054, but **WE ARE DOING A LOWER-TAILED TEST! THE z^* CRITICAL VALUE IS NEGATIVE! z^* = –2.054**.

$$\bar{x}^* = z^* \frac{\sigma}{\sqrt{n}} + \mu_0 = -2.054 \times \frac{20}{\sqrt{64}} + 100 = \mathbf{94.865}$$

Solution to Question 3 (d)

The $\bar{x}$ decision rule is "Reject H_0 if $\bar{x}$ < 94.865."

Of course, once you know when you will reject H_0, you also know when you will not reject H_0. For example, if you will "reject H_0 if $\bar{x} > 100$", then you know you "do not reject H_0 if $\bar{x} \leq 100$" (the exact opposite region). **Recall, as I discussed at the start of lesson 2, the opposite of ">" is "≤"; one of the two statements must include the equals sign.** If you are doing a two-tailed test, you might have an $\bar{x}$ decision rule like "reject H_0 if $\bar{x} < 25$ or $\bar{x} > 42$". Visualize those two tails shaded on a bell curve, one shading to the left of 25, the other to the right of 42. That means you won't reject H_0 if $\bar{x}$ is anywhere between 25 and 42. To say $\bar{x}$ is between 25 and 42 mathematically, simply put $\bar{x}$ between the two numbers (smaller number on the left) and use "<" signs or "≤" signs, depending on whether you need to include the endpoints themselves (never use ">" or "≥" signs when you are trying to say $\bar{x}$ is between two numbers). Thus, if we "reject H_0 if $\bar{x} < 25$ or $\bar{x} > 42$", then we "do not reject H_0 if $25 \leq \bar{x} \leq 42$".

Here are some examples:

When we reject H_0	When we do not reject H_0
Reject H_0 if $\bar{x} > 65$	Do not reject H_0 if $\bar{x} \leq 65$
Reject H_0 if $\bar{x} < 98$	Do not reject H_0 if $\bar{x} \geq 98$
Reject H_0 if $\bar{x} < 17$ or $\bar{x} > 19$	Do not reject H_0 if $17 \leq \bar{x} \leq 19$

THE ALTERNATIVE MEAN, μ_a

If we want to compute P(Type II error) or the power of a hypothesis test for μ, we also must have an **alternative value for the mean** which I denote $\boldsymbol{\mu_a}$, just like we use H_a to denote the alternative hypothesis; I have also seen some profs denote the alternative mean as μ^*, but I prefer to use "*" only when denoting critical values. The reason we need an alternative mean is both β and power believe the null hypothesis, H_0, is wrong.

Recall: a Type II error is "not rejecting H_0 when H_0 is false," and power is the probability of correctly rejecting H_0 (which again implies H_0 is false). Since H_0 is false, μ_0, the value H_0 believes the mean to be, must be wrong. But, if μ_0 is wrong, then what is the correct value for μ? That's where μ_a comes in. **We must have an alternative mean, $\boldsymbol{\mu_a}$, in order to be able to compute $\boldsymbol{\beta}$ or power in a hypothesis test for the mean, $\boldsymbol{\mu}$.**

THE α/β TABLE

The easiest way to compute α, β or power is to organize all the given information into what I call the "α/β Table". After you have noted all the givens for the problem, **make sure you know the $\bar{x}$ decision rule**. Sometimes you have been given that already. If not, use "$\bar{x}^* = z^* \frac{\sigma}{\sqrt{n}} + \mu_0$" to determine the $\bar{x}$ decision rule yourself.

Do note: In order to compute these probabilities we will be assuming it is valid to draw an $\bar{x}$-bell curve (which is to say, we are assuming $\bar{x}$ has a normal distribution). As I have said a million times already, be very aware of the importance of having a normal distribution. Normal distributions enable us to use z, and z enables us to determine probabilities (thanks to Table A*).

Again, if we know a population is normal (because they told us so), then we also know $\bar{x}$ has a normal distribution (thus it is valid to use an $\bar{x}$-bell curve). If we are not told a population is normal, we can still assume $\bar{x}$ is *approximately* normal, as long as n is large (we want $n = 15$ at least, but the larger the better).

If we can assume $\bar{x}$ is normally distributed and draw an $\bar{x}$-bell curve, and if we are given σ, the population standard deviation, then we can compute z and use Table A to find probabilities. You can safely bet all these conditions will be met in any problem that asks us to compute α, β or power.

Recall, the z standardizing formula for the $\bar{x}$-bell curve is $\boldsymbol{z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}}}$.

Note: I have used "μ" in the standardizing formula above, not μ_0 as used in the z test statistic formula. This is because we should always use the *true* value for μ when computing z. In hypotheses, we always assume the null hypothesis is correct, thus we use μ_0 when computing the test statistic. But, when we are computing β, we believe H_0 is false, so we would never use μ_0 to compute z. Instead we will use μ_a. Pay very close attention to the steps I outline below.

* I am referring to "Table A: Standard normal probabilities" in Moore & McCabe's *Introduction to the Practice of Statistics*. This table will be used throughout this book to find probabilities for associated z-scores.

TO CONSTRUCT THE "α/β TABLE" AND COMPUTE THE z-SCORE:

Step 1. **Label the first column "α" and the second column "β".** The "α" column will list all the info we need to compute $\alpha = P$(Type I error) while the "β" column will list all the info we need to compute $\beta = P$(Type II error). The "β" column will also enable us to compute the power since it is $1 - \beta$.

Step 2. **In the "α" column, list when we will reject H_0 according to the $\bar{x}$ decision rule. In the "β" column, list when we will not reject H_0 according to the $\bar{x}$ decision rule.**

Step 3. **In the "α" column, list what μ_0 equals (as stated by H_0). In the "β" column, list what μ_a equals (as given).**

Step 4. **In the "α" column, you can now draw an $\bar{x}$-bell curve illustrating all the info listed in that column.** (The $\bar{x}$-bell curve will be centred at **the hypothesized mean μ_0** and you will mark and shade the $\bar{x}$ values where you will reject H_0 according to the $\bar{x}$ decision rule. **That shaded area represents α.**)

In the "β" column, you can draw an $\bar{x}$-bell curve illustrating all the info listed in that column. (The $\bar{x}$-bell curve will be centred at **the alternative mean μ_a** and you will mark and shade the $\bar{x}$ values where you will not reject H_0 according to the $\bar{x}$ decision rule. **That shaded area represents β.)**

Step 5. We then use " $z = \dfrac{\bar{x} - \mu}{\sigma/\sqrt{n}}$ " to compute the z-score and get the probability from Table A. **Note: the "α" column will use μ_0** since that is what the "α" column believes to be the true mean while **the "β" column, will use μ_a** since that is what the "β" column believes to be the true mean.)

4. You have an SRS (simple random sample) of size n = 16 from a normal distribution with $\sigma = 6$. You wish to test H_0: $\mu = 12$ vs. H_a: $\mu > 12$, and you decide to reject H_0 if $\bar{x} > 15$.

(a) What is the probability of a Type I error?

(A) 0.0228 (B) 0.05 (C) 0.7486 (D) 0.4987 (E) 0.7734

(b) If $\mu = 16$ in fact, what is the probability of a Type II error?

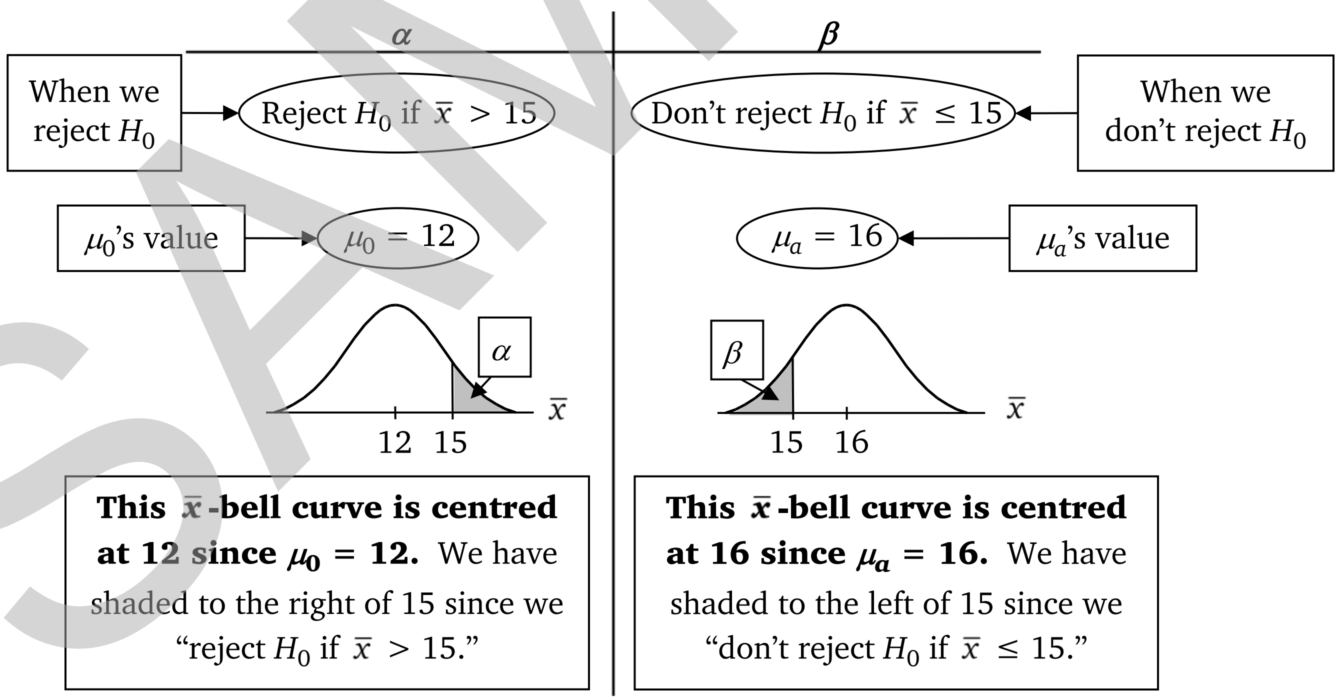
(A) 0.0013 (B) 0.7266 (C) 0.7486 (D) 0.2734 (E) 0.2514

(c) If $\mu = 16$ in fact, what is the power of the test?

(A) 0.9987 (B) 0.7266 (C) 0.7486 (D) 0.2734 (E) 0.2514

<u>Givens</u>: n = 16; normal distribution (we can draw an $\bar{x}$-bell curve); **σ = 6** (we can use z); **H_0: μ = 12** (we are given μ_0 = 12) vs. **H_a: $\mu > 12$** (an upper-tailed test); **reject H_0 if $\bar{x} > 15$** (we have been given the $\bar{x}$ decision rule). Parts (b) and (c) give us an alternative value for μ (**μ_a = 16**) which is necessary if we are to compute β or power.

Construct the "α/β Table".



How does the "α/β Table" work?

We are simply using a very mechanical way to write down exactly what is needed to work out the probabilities of Type I error and Type II error. Recall: A Type I error is "rejecting H_0 when H_0 is true". Well, if we look at the "α" column of the "α/β Table" I just constructed for **question 4**, the $\bar{x}$-bell curve I drew is centred at μ_0 = 12 (because H_0 said μ = 12, and this bell curve believes H_0 is true). But, I have shaded the region where $\bar{x}$ > 15 (which is the values of $\bar{x}$ where we will reject H_0). Which is to say, **the "α" $\bar{x}$-bell curve is shading the region where we will reject H_0 even if H_0 is true**; thus, that area is α, the probability of a Type I error.

$$\boxed{\alpha = P(\underbrace{\text{rejecting } H_0}_{\bar{x}>15} \text{ when } \underbrace{H_0 \text{ is true}}_{\mu=12})}$$

Similarly, a Type II error is "not rejecting H_0 when H_0 is false". Well, if we look at the "β" column of the "α/β Table", the $\bar{x}$-bell curve I drew is centred at μ_a = 16 (because H_0 said μ = 12 but we believe H_0 to be <u>false</u>; instead we believe μ = 16, the alternative mean we were given; this bell curve centred at 16 says H_0 is false). But, I have shaded the region where $\bar{x}$ ≤ 15 (which is the values of $\bar{x}$ where we will <u>not</u> reject H_0). Which is to say, **the "β" $\bar{x}$-bell curve is shading the region where we will <u>not</u> reject H_0 even though H_0 is false**; thus, that area is β, the probability of a Type II error.

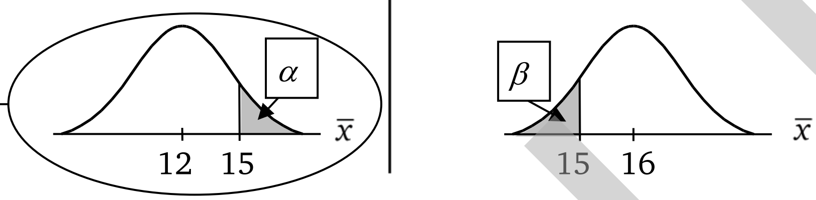
$$\boxed{\beta = P(\underbrace{\text{not rejecting } H_0}_{\bar{x}\le 15} \text{ when } \underbrace{H_0 \text{ is false}}_{\mu=16})}$$

But, that is the beauty of the "α/β Table"; we don't have to think about any of that stuff. We simply fill in the $\bar{x}$ decision rule. We put when we reject H_0 on the "α" side and when we don't reject H_0 on the "β" side. Then we put μ_0 on the "α" side and μ_a on the "β" side. Finally, we draw an $\bar{x}$-bell curve illustrating these statements, and we have quickly and easily visualized precisely what area must be found to determine the values of α and β.

4. (a) What is the probability of a Type I error?

(A) 0.0228 (B) 0.05 (C) 0.7486 (D) 0.4987 (E) 0.7734

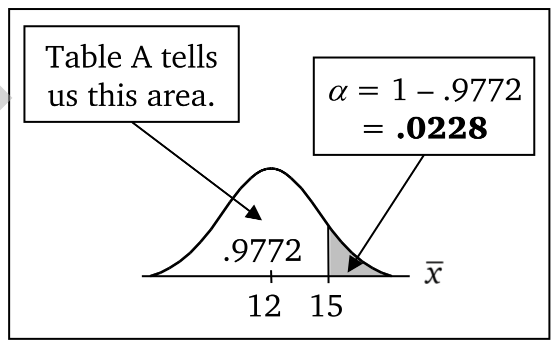
α	β
Reject H_0 if $\bar{x} > 15$	Don't reject H_0 if $\bar{x} \le 15$
$\mu_0 = 12$	$\mu_a = 16$



P*(Type I error) = *α, which the "α" column of our "α/β Table" shows is the area to the right of 15, assuming $\mu = 12$. Use the $\bar{x}$ standardizing formula to compute a z-score.

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{15 - 12}{6/\sqrt{16}} \rightarrow \boldsymbol{z = 2}$$

According to Table A, when $z = 2$, the left area is .9772 (which is, therefore, the area to the left of 15 on our $\bar{x}$-bell curve). We want the right area, however, so ***P*(Type I error) = *α* = 1 – .9772 = .0228**.



Solution to Question 4 (a)

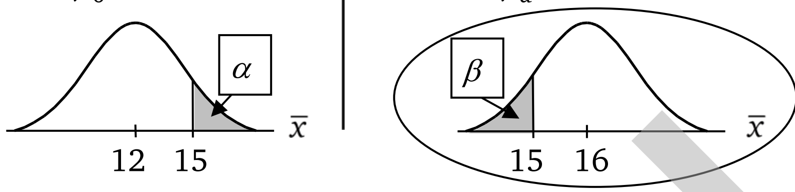
The probability of Type I error is .0228. The correct answer is (A).

Don't make the mistake of thinking, "Since α isn't given, shouldn't we say α = 5%?" Rest assured, if you are asked to determine the probability of Type I error, and α has not been given, then you must use the "α/β Table" to compute it yourself. Which is to say, they <u>did</u> choose a value for α, they just aren't telling you what it is. Of course, if α is given, they have handed you the probability of Type I error, no work necessary.

4. (b) If μ = 16 in fact, what is the probability of a Type II error?

(A) 0.0013 (B) 0.7266 (C) 0.7486 (D) 0.2734 (E) 0.2514

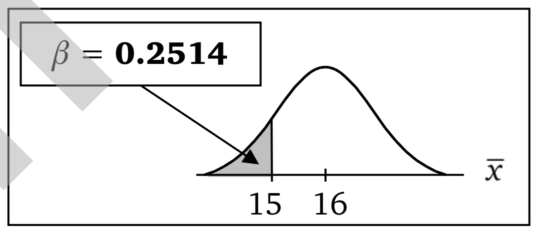
α	β
Reject H_0 if $\bar{x} > 15$	Don't reject H_0 if $\bar{x} \le 15$
$\mu_0 = 12$	$\mu_a = 16$



***P*(Type II error) = β**, which the "β" column of our "α/β Table" shows is the area to the left of 15, assuming μ = 16. Use the $\bar{x}$ standardizing formula to compute a z-score.

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{15 - 16}{6/\sqrt{16}} \rightarrow \boldsymbol{z = -0.67}$$

According to Table A, when z = –0.67, the left area is .2514 (which is, therefore, the area to the left of 15 on our $\bar{x}$-bell curve). This is precisely the tail area we want, so ***P*(Type II error) = β = .2514**.



Solution to Question 4 (b)

The probability of Type II error is .2514. The correct answer is (E).

4. (c) If μ = 16 in fact, what is the power of the test?

(A) 0.9987 (B) 0.7266 (C) 0.7486 (D) 0.2734 (E) 0.2514

We know **power = 1 – β**, and we just found β = .2514 in part (b). Therefore, **power = 1 – .2514 = .7486**.

Solution to Question 4 (c)

The power is .7486. The correct answer is (C).

5. You are examining a normal population with σ = 10. You wish to test H_0: μ = 30 vs. H_a: $\mu \neq$ 30 by taking a random sample of size 25. You decide to reject H_0 if $\bar{x}$ < 27 or $\bar{x}$ > 33.

(a) What is the probability of a Type I error?

(A) 0.0668 (B) 0.0548 (C) 0.1336 (D) 0.1096 (E) 0.05

(b) If μ = 28, in fact, what is the power of the test?

(A) 0.3147 (B) 0.9332 (C) 0.05 (D) 0.6853 (E) 0.8664

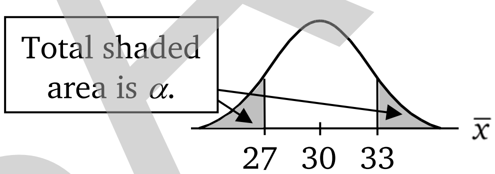
(c) If μ = 25, in fact, what is the power of the test?

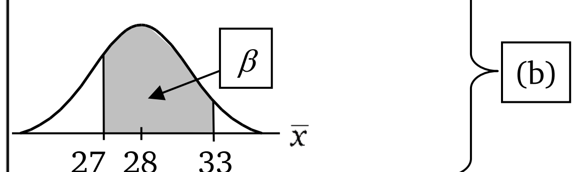
(A) 0.1587 (B) 0.8849 (C) 0.8907 (D) 0.1093 (E) 0.8413

<u>Givens</u>: normal population (we can draw an $\bar{x}$-bell curve); **σ = 10** (we can use z); **H_0: μ = 30** (we are given **μ_0 = 30**) vs. **H_a: $\mu \neq$ 30** (a two-tailed test); **n = 25**; **reject H_0 if $\bar{x}$ < 27 or $\bar{x}$ > 33** (we have been given the $\bar{x}$ decision rule). Note that parts (b) and (c) give us alternative values for μ (**μ_a = 28** in part (b) and **μ_a = 25** in part (c)) which is necessary if we are to compute β or power.

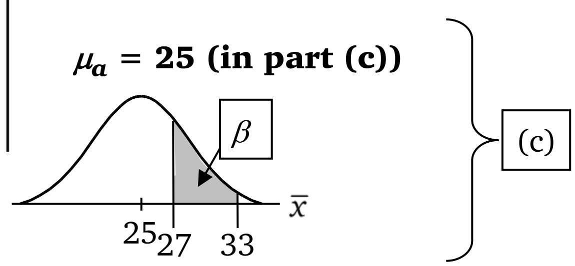
Construct the "α/β Table" (note, I am drawing two separate pictures on the "β" side to represent the situation for the two different values of μ_a).

α	β
Reject H_0 if $\bar{x}$ < 27 or $\bar{x}$ > 33	Don't reject H_0 if 27 ≤ $\bar{x}$ ≤ 33
μ_0 = 30	**μ_a = 28 (in part (b))**





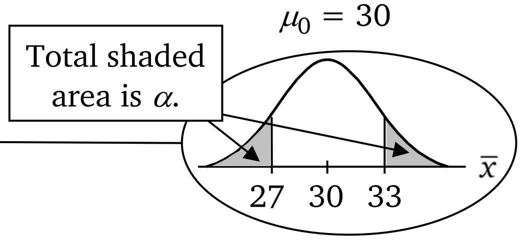
μ_a = 25 (in part (c))



5. (a) What is the probability of a Type I error?

(A) 0.0668 (B) 0.0548 (C) 0.1336 (D) 0.1096 (E) 0.05

α	β
Reject H_0 if $\bar{x} < 27$ or $\bar{x} > 33$	**IRRELEVANT**
$\mu_0 = 30$	

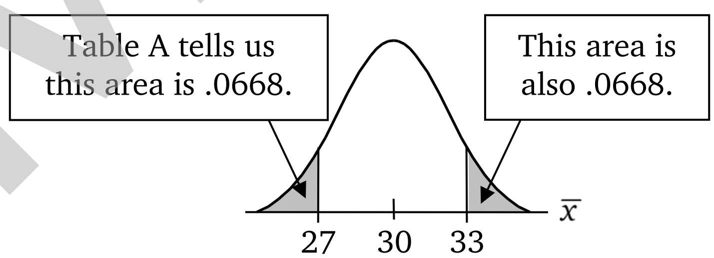


***P*(Type I error) = α**, which the "α" column of our "α/β Table" shows is the combined area of the two tails bounded by 27 and 33, respectively, assuming $\mu = 30$. Use the $\bar{x}$ standardizing formula to compute the two z-scores.

When $\bar{x} = 27$: $z = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \dfrac{27-30}{10/\sqrt{25}} \rightarrow \mathbf{z = -1.5}$

When $\bar{x} = 33$: $z = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \dfrac{33-30}{10/\sqrt{25}} \rightarrow \mathbf{z = 1.5}$

According to Table A, when $z = -1.5$, the left area is .0668, giving us the lower tail. Since $z = 1.5$ is symmetrical, the upper tail also has an area of .0668, so the total shaded area $= \alpha =$ **.0668 × 2 = .1336**.



***P*(Type I error) = α = .0668 × 2 = .1336**

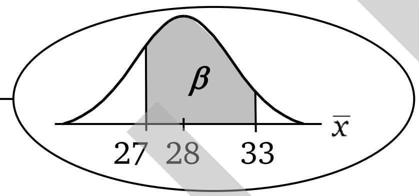
Solution to Question 5 (a)

The probability of Type I error is .1336. The correct answer is (C).

5. (b) If μ = 28, in fact, what is the power of the test?

(A) 0.3147 (B) 0.9332 (C) 0.05 (D) 0.6853 (E) 0.8664

α	β
IRRELEVANT	Don't reject H_0 if $27 \le \bar{x} \le 33$
	$\mu_a = 28$



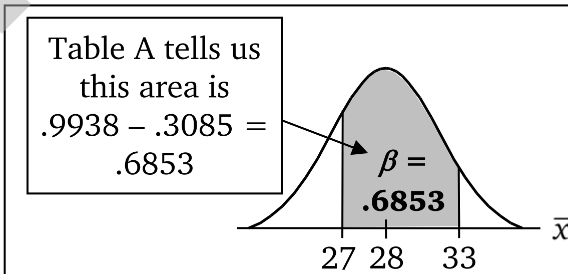
Power = 1 – β, and the "β" column of our "α/β Table" shows that β is the area between 27 and 33, assuming μ = 28. Use the $\bar{x}$ standardizing formula to compute the two z-scores.

When $\bar{x}$ = 27: $z = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \dfrac{27-28}{10/\sqrt{25}} \rightarrow$ $\boldsymbol{z = -0.5}$**, Left Area = .3085**

When $\bar{x}$ = 33: $z = \dfrac{\bar{x}-\mu}{\sigma/\sqrt{n}} = \dfrac{33-28}{10/\sqrt{25}} \rightarrow$ $\boldsymbol{z = 2.5}$**, Left Area = .9938**

According to Table A, when z = –0.5, the left area is .3085. When z = 2.5, the left area is .9938. Therefore, the shaded area = ***P*(Type II error) = β = .9938 – .3085 = .6853.*** However, it is the power we want.

Power = 1 – β = 1 – .6853 = .3147.



Power = 1 – β = 1 – .6853 = .3147

Solution to Question 5 (b)

The power of the test when μ = 28, in fact, is .3147.
The correct answer is (A).

* Table A always gives you the area to the left of any z-score. If you ever want to find the area between two z-scores, find the left area for each one, and subtract.

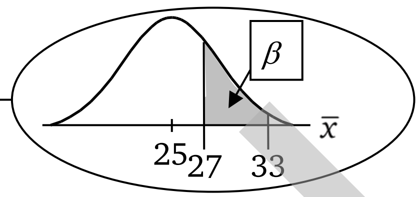
The "between area" = the difference between the two left areas.

5. (c) If $\mu = 25$, in fact, what is the power of the test?

(A) 0.1587 (B) 0.8849 (C) 0.8907 (D) 0.1093 (E) 0.8413

α	β
IRRELEVANT	Don't reject H_0 if $27 \le \bar{x} \le 33$

$\boldsymbol{\mu_a = 25}$

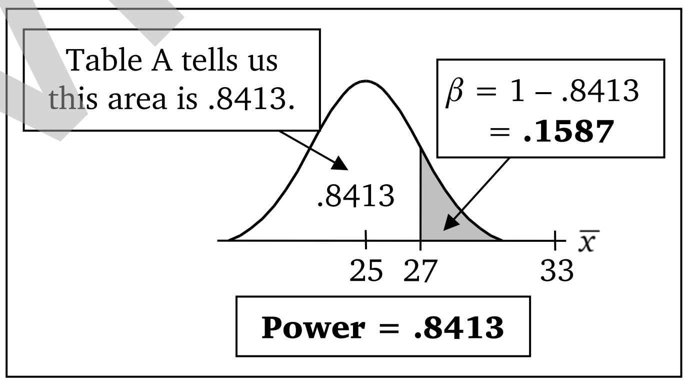


Power = 1 – $\boldsymbol{\beta}$, and the "β" column of our "α/β Table" shows that β is the area between 27 and 33, assuming $\mu = 25$. Use the $\bar{x}$ standardizing formula to compute the two z-scores.

When $\bar{x} = 27$: $z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \dfrac{27 - 25}{10 / \sqrt{25}} \rightarrow \boldsymbol{z = 1}$

When $\bar{x} = 33$: $z = \dfrac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \dfrac{33 - 25}{10 / \sqrt{25}} \rightarrow \boldsymbol{z = 4}$

When we consult Table A, we discover $z = 4$ isn't even on the table! **Whenever a $\boldsymbol{z}$-score is not on Table A, this indicates that particular value is <u>off the end of the bell curve</u>. Redraw your picture showing that score off the end to better visualize the area you are after.** In this case, 33 is so far off to the right it becomes irrelevant to the problem.



When $z = 1$, the left area is .8413. This must be the power! Think about it. If the shaded area is β, then the rest of the curve (the unshaded part) is "$1 - \beta$", which is the power. **Power = .8413.** (This checks out. Since β is the right area in our diagram, $\beta = 1 - .8413 = .1587$ (since right area = 1 – left area). If $\beta = .1587$, then power $= 1 - \beta = 1 - .1587 = .8413$, as predicted.)

Solution to Question 5 (c)

The power of the test when $\mu = 25$, in fact, is .8413.
The correct answer is (E).

On the "β" side of our "α/β Table", we always shade the area that represents β. Consequently, the unshaded area must be the power (since the total area is 1). Keeping in mind Table A always gives us the left area for any *z*-score we cross-reference, sometimes it is easier to get β first, then compute "1 – β" to get the power. Other times, it is easier to get the power first, then compute "1 – power" to get β. Draw a diagram, visualize the area Table A is giving you, and plan your strategy to get the area you desire.

<u>Simply put</u>: What we shade on the "β" side of our "α/β Table" always represents β. The <u>unshaded</u> part, therefore, is the power. Table A always gives us the Left Area. Sometimes that Left Area is the power; sometimes that Left Area is β; sometimes it is neither. In any case, we always know β and power add up to 1 or 100%.

<u>Note</u>: In question 5, we were given $\mu_0 = 30$. When $\mu_a = 28$, as given in part (b), the power = .3147 (very poor); when $\mu_a = 25$, as given in part (c), the power = .8413 (better). **As the alternative mean μ_a moves farther away from the hypothesized mean μ_0, the power increases.**

This makes sense. Since power is the probability of correctly rejecting H_0, the more different μ_a is from μ_0, the more likely it is we should be able to spot that H_0 is wrong.

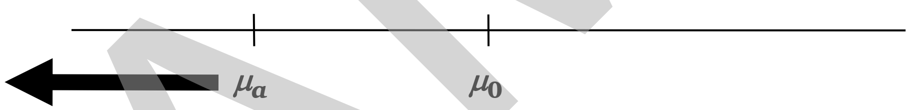
THE RELATIONSHIP BETWEEN μ_a AND THE POWER OF A TEST

It might help to visualize this on a number line. Place the value of μ_0 on the line. Then, place the different values of μ_a you are given on the line. The farther along the number line μ_a is from μ_0 (in the appropriate direction), the higher the power of the test will be.

If you are doing an <u>upper</u>-tailed test, then you will expect μ_a to be to the <u>right</u> of μ_0 (because you believe μ is <u>larger</u> than the hypothesized value). The farther to the right μ_a is from μ_0, the larger the power of the test will be.

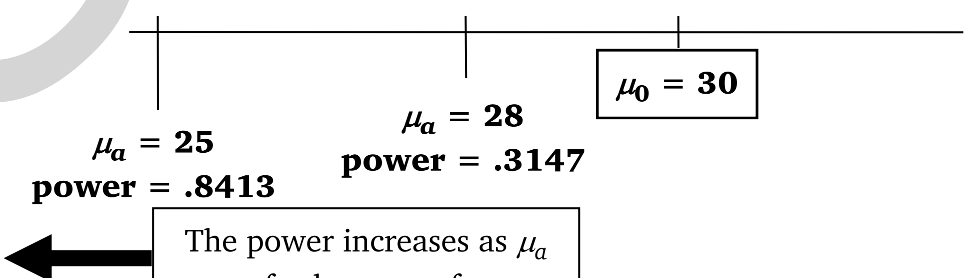
The power increases as μ_a moves farther away from μ_0.

If you are doing a <u>lower</u>-tailed test, then you will expect μ_a to be to the <u>left</u> of μ_0 (because you believe μ is <u>smaller</u> than the hypothesized value). The farther to the left μ_a is from μ_0, the larger the power of the test will be.



The power increases as μ_a moves farther away from μ_0.

In question 5, we are doing a two-tailed test, so it doesn't matter what side μ_a is on in relation to μ_0 (we believe μ could be smaller or larger than the hypothesized value).



Since **question 5** is a two-tailed test, the alternative means could just as easily have been on the right of μ_0. We know from part (b) power = .3147 when μ_a = 28, which is 2 units below μ_0 = 30. Just as power increases as we get farther away from μ_0, it also follows that **power will be the same if we are equidistant from μ_0.** μ_a = 32 is also 2 units from μ_0 = 30, and so must have the same power as μ_a = 28. We can exploit symmetry to say, **if μ_a = 32, the power is .3147 (the same power as for μ_a = 28)**. Similarly, μ_a = 25 and μ_a = 35 are both 5 units away from μ_0 = 30, so they must have the same power. Thus, **if μ_a = 35, the power is .8413 (the same power as for μ_a = 25)**.

Only for two-tailed tests will the power for alternative means equidistant from μ_0 be the same. If we were doing a lower-tailed test in question 5, then μ_a = 32 or μ_a = 35 would make no sense. The alternative hypothesis would be saying the mean is lower than 30 (H_a: $\mu < 30$), so the alternative means must be lower than 30 (like 28 or 25). There is no way we could say the power = .3147 for μ_a = 32; μ_a would have no business being 32! If that were the case, H_a: $\mu < 30$ would have been wrong (since μ = 32 is certainly not less than 30), rendering power meaningless (we shouldn't be rejecting H_0, so the power, the probability of *correctly* rejecting H_0, is 0).

Simply put:

> **For lower-tailed tests, μ_a must be lower than μ_0.**
>
> **For upper-tailed tests, μ_a must be higher than μ_0.**
>
> **For two-tailed tests, μ_a can be lower or higher than μ_0.**
>
> **In all cases, as long as μ_a is on the correct side of μ_0, the farther μ_a gets from μ_0, the higher the power will be.**

6. A sample of size 15 is selected from a normal population with variance 100. The null hypothesis H_0: $\mu = 77$ is tested against the alternative hypothesis H_a: $\mu > 77$ at the 5% level.

(a) What is the probability of a Type II error if μ is actually 83?

(A) 0.2517 (B) 0.7483 (C) 0.2483 (D) 0.7517 (E) 0.0500

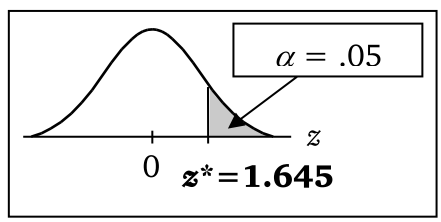
(b) What is the probability of a Type I error?

(A) 0.2517 (B) 0.7483 (C) 0.2483 (D) 0.7517 (E) 0.0500

<u>**Givens**</u>**:** $\boldsymbol{n} = \mathbf{15}$; **normal population** (we can draw an $\bar{x}$-bell curve); $\boldsymbol{\sigma^2} = \mathbf{100}$, so $\boldsymbol{\sigma = \sqrt{100} = 10}$ (we can use z); $\boldsymbol{H_0}$**:** $\boldsymbol{\mu = 77}$ (we are given $\boldsymbol{\mu_0} = 77$) vs. $\boldsymbol{H_a}$**:** $\boldsymbol{\mu > 77}$ (an upper-tailed test); $\boldsymbol{\alpha} = \mathbf{5\%}$; in part (a) of the question we are given $\boldsymbol{\mu_a} = \mathbf{83}$.

We are <u>not</u> given the $\bar{x}$ decision rule. There is no way we can compute α, β or power without it! So, first and foremost, use $\boldsymbol{\bar{x}^* = z^* \frac{\sigma}{\sqrt{n}} + \mu_0}$ to compute the critical $\bar{x}$ value and state the decision rule. Also, note this formula uses μ_0, not μ_a! **We will often be given both μ_0 and μ_a in these kinds of questions, but be sure to use μ_0 to compute $\bar{x}^*$.**

First we need to know z^*, the critical z value. We get z^* the way we usually do in hypothesis testing (as we learned in Lesson 2). We are conducting an **upper-tailed test at $\alpha = 5\%$**, so visualize 5% shaded in the upper tail of the z-bell curve. Consulting Table D at the ".05" column, we see $\boldsymbol{z^* = 1.645}$.



$$\bar{x}^* = z^* \frac{\sigma}{\sqrt{n}} + \mu_0 = 1.645 \times \frac{10}{\sqrt{15}} + 77 = \mathbf{81.24737...}$$

To ensure you do not lose accuracy which may affect your final answer, **never round off to less than 4 decimal places** in any computations you make. Better yet, avoid rounding off at all by storing messy answers in the memory of your calculator.

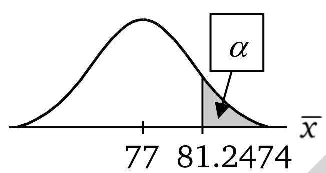
Now that we know $\boldsymbol{\bar{x}^* = 81.2474}$, our upper-tailed $\bar{x}$ decision rule is **reject H_0 if $\bar{x} > 81.2474$**. Now we are ready to answer the questions.

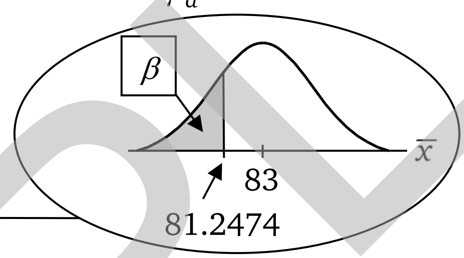
6. (a) What is the probability of a Type II error if μ is actually 83?

(A) 0.2517 (B) 0.7483 (C) 0.2483 (D) 0.7517 (E) 0.0500

In this problem, the "α" side of the "α/β Table" is irrelevant.
I am filling it in just for practise, but you could choose to omit it altogether.

α	β
Reject H_0 if $\bar{x} > 81.2474$	Don't reject H_0 if $\bar{x} \le 81.2474$
$\mu_0 = 77$	$\mu_a = 83$

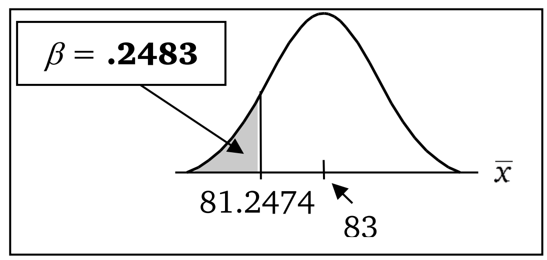




P*(Type II error) = *β, which the "β" column of our "α/β Table" shows is the area to the left of 81.2474, assuming $\mu = 83$. Use the $\bar{x}$ standardizing formula to compute a z-score.

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{81.2474 - 83}{10/\sqrt{15}} \rightarrow \boldsymbol{z = -0.68}$$

According to Table A, when $z = -0.68$, the left area is .2483. β is this tail area, so ***P*(Type II error) = *β* = .2483**.



Solution to Question 6 (a)

The probability of Type II error is .2483. The correct answer is (C).

6. (b) What is the probability of a Type I error?

(A) 0.2517 (B) 0.7483 (C) 0.2483 (D) 0.7517 (E) 0.0500

Piece o' cake! ***P*(Type I error) = *α***, which we were told is 5%!

Solution to Question 6 (b)

The probability of Type I error is 5% = .05. The correct answer is (E)

7. A researcher believes a company's claim its batteries last an average of at least 100 hours is bogus. He intends to take a random sample of 64 batteries and will reject their claim if $z < -2$. He assumes that the population has a standard deviation of 10 hours.

(a) What is a Type I error in this context?

(b) What is a Type II error in this context?

(c) What level of significance is the researcher using?

(d) If the true mean is 90 hours, then what is the power of this test?

(e) If the true mean is 90 hours, is this a reliable test?

(f) If the true mean is 98 hours, is this a reliable test?

Givens: The batteries last "*an average of at least* 100 hours" ($\mu \geq 100$). The opposite side of the argument is $\mu < 100$. H_0 will take "$\mu \geq 100$" since that includes the equals sign and H_a will take the "$\mu < 100$". So, we can formulate the hypotheses $\boldsymbol{H_0}$**:** $\boldsymbol{\mu = 100}$ ($\boldsymbol{\mu_0 = 100}$) vs. $\boldsymbol{H_a}$**:** $\boldsymbol{\mu < 100}$ (a lower-tailed test). We are also given $\boldsymbol{n = 64}$ and told to **reject** $\boldsymbol{H_0}$ **if** $\boldsymbol{z < -2}$ which tells us the z critical value is $\boldsymbol{z^* = -2}$. We have been given a lower-tailed z decision rule which makes sense since we have established we want to do a lower-tailed test. We are also given $\boldsymbol{\sigma = 10}$ (which explains why the researcher can use z). We have no idea whether the population is normal or not, but n is large, so we can assume $\bar{x}$ is normally distributed (we can draw an $\bar{x}$-bell curve). Finally, in part (d) of the question we are given $\boldsymbol{\mu_a = 90}$.

Be careful! Parts (a) and (b) of this question are not asking for the *probabilities* of Type I and Type II error (they are not asking for α and β). They are simply asking us what the errors are in this context.

7. (a) What is a Type I error in this context?

We have determined the hypotheses to be $\boldsymbol{H_0}$**:** $\boldsymbol{\mu = 100}$ **vs.** $\boldsymbol{H_a}$**:** $\boldsymbol{\mu < 100}$.

A Type I error is rejecting H_0 when H_0 is true. In this context, that would mean:

$$\underbrace{\text{rejecting } H_0}_{\substack{\text{concluding} \\ \mu < 100}} \text{ when } \underbrace{H_0 \text{ is true}}_{\mu = 100}$$

Solution to Question 7 (a)

A Type I error in this context is concluding the batteries last an average of less than 100 hours when, in fact, they last 100 hours (at least), as the company claims. (Put another way: The Type I error is to conclude the company's claim is "bogus" when it is not.)

7. (b) What is a Type II error in this context?

A Type II error is <u>not</u> rejecting H_0 when H_0 is false. In this context, that would mean:

$$\underbrace{\text{not rejecting } H_0}_{\substack{\text{concluding} \\ \mu = 100}} \text{ when } \underbrace{H_0 \text{ is false}}_{\mu < 100}$$

Solution to Question 7 (b)

A Type II error in this context is concluding the batteries last an average of at least 100 hours (as the company claims) when, in fact, they last less than 100 hours. (Put another way: The Type II error is to conclude the company's claim is <u>not</u> "bogus" when it is.)

7. (c) What level of significance is the researcher using?

The "level of significance" is α. But we weren't given α! What we must realize is this question is really asking us to determine the probability of Type I error because $\alpha = P(\text{Type I error})$. **Asking for the level of significance is the same thing as asking for the probability of Type I error because they are both α.** Reading the whole problem should have got us thinking about α, β and power and constructing the "α/β Table", anyway.

We are not given the $\bar{x}$ decision rule. There is no way we can compute α, β or power without it! So, first and foremost, use $\bar{x}^* = z^* \frac{\sigma}{\sqrt{n}} + \mu_0$ to compute the critical $\bar{x}$ value and state the decision rule.

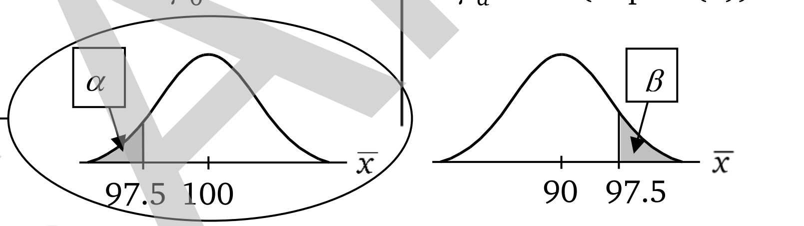
First we need to know z^*, the critical z value. We were "given" $z^* = -2$. (The z decision rule said we will "reject their claim if $z < -2$", so -2 must be the critical value.)

$$\bar{x}^* = z^* \frac{\sigma}{\sqrt{n}} + \mu_0 = -2 \times \frac{10}{\sqrt{64}} + 100 \;\rightarrow\; \bar{x}^* = 97.5$$

So, our lower-tailed $\bar{x}$ decision rule is **reject H_0 if $\bar{x} < 97.5$**.

Now we are ready to make the "α/β Table" and answer the rest of question 6.

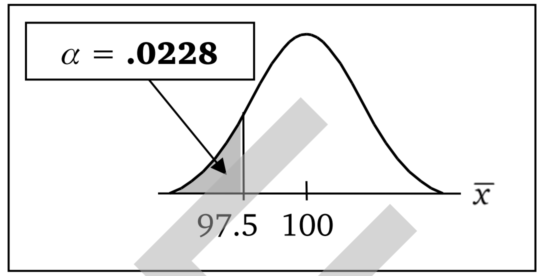
α	β
Reject H_0 if $\bar{x} < 97.5$	Don't reject H_0 if $\bar{x} \geq 97.5$
$\mu_0 = 100$	$\mu_a = 90$ (in part (d))



***P*(Type I error) = α**, which the "α" column of our "α/β Table" shows is the area to the left of 97.5, assuming $\mu = 100$. Use the $\bar{x}$ standardizing formula to compute a z-score.

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{97.5 - 100}{10/\sqrt{64}} \;\rightarrow\; z = -2$$

According to Table A, when $z = -2$, the left area is .0228. Perfect! This is precisely the tail area we want, so **α = *P*(Type I error) = the level of significance = .0228.**[*]

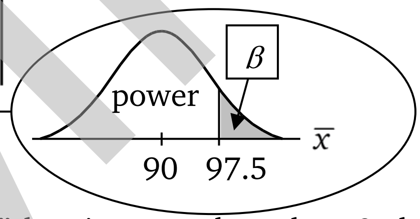


Solution to Question 7 (c)

The probability of Type I error, and so also α, the level of significance, is .0228.

7. (d) If the true mean is 90 hours, then what is the power of this test?

α	β
Reject H_0 if $\bar{x} < 97.5$	Don't reject H_0 if $\bar{x} \geq 97.5$
$\mu_0 = 100$	$\mu_a = 90$

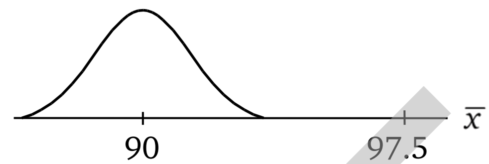


The "β" column of our "α/β Table" has instructed us that β, the probability of Type II error, is the area to the right of 97.5, assuming the true mean is 90. However, this question has asked us to find power. Power is the unshaded region on this bell curve (since β and power add up to 1). **Thus, in this problem, the power is the left area on the bell curve!** Use the $\bar{x}$ standardizing formula to compute the z-score.

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{97.5 - 90}{10/\sqrt{64}} \rightarrow \boldsymbol{z = 6}$$

[*] If you think about it, we were told at the start we will reject H_0 if $z < -2$ so it makes sense that α is the area to the left of $z = -2$. After all, that is how we come up with rejection regions in the first place: we shade the area "α" and get the associated critical value z^*. Instead of being given α and using that to get z^*, we have been given z^* and have been asked to find α. No harm done. We needed to compute the $\bar{x}$ decision rule to get ready to solve the rest of the parts in this question anyway.

But, $z = 6$ is way off the end of Table A. That tells us 97.5 must be way off the end of the $\bar{x}$-bell curve. Redraw the bell curve to take this into account and better visualize the β and power areas. Clearly, we can see the power, the area to the left of 97.5, is 1 (the entire bell curve), and β, the area to the right of 97.5, is so small it is, for all practical purposes, 0. Thus, $\boldsymbol{\beta} = \mathbf{0}$ and **power = 1.***



Since $z = 6$, we know 97.5 is way off to the right side of the curve. Clearly, the tail area is so small it must be 0. Thus, $\boldsymbol{\beta} = \mathbf{0}$ and **power = 1**.

Solution to Question 7 (d)

If the true mean is 90 hours, the power of the test is 1.

7. (e) If the true mean is 90 hours, is this a reliable test?

A reliable test is one that you can *rely* upon to make correct decisions. Which is to say, if you should reject H_0 (because H_0 is wrong), you want to be confident that your test will tell you to reject H_0. Similarly, if you should not reject H_0 (because H_0 is correct), you want to be confident that your test will not allow you to reject H_0.

Any hypothesis test that has a low probability of error (low α and low β), and a high power is a <u>reliable</u> test.

Solution to Question 7 (e)

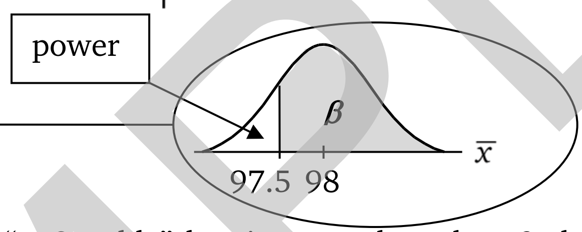
We have found the power is 1 if the true mean is 90 hours. There is essentially a 100% chance that this experiment can be relied upon to spot the mean is <u>not</u> "at least 100 hours", as the company claims, if the true mean is 90 hours. This is a very reliable test.

* If you want to be technical, the highest area we can find on Table A is .9998 (when $z = 3.49$); anytime we get a z-score that is off the end of the curve (in either direction), the major area is certain to be larger than .9998, and the tail area is smaller than .0002. In this particular case, we can say "the power > .9998" and "β < .0002". However, it is fine to say the area power = 1 and $\beta = 0$.

7. (f) If the true mean is 98 hours, is this a reliable test?

To assess whether or not this test is reliable if the true mean is 98 hours, we will have to compute β and/or the power. They have not changed the z decision rule, so that means our $\bar{x}$ bar decision rule is still the same as well. We still **reject H_0 if $\bar{x} < 97.5$**. However, we now have a new alternative mean, $\boldsymbol{\mu_a = 98}$. Construct a new "α/β Table" and compute the new power.

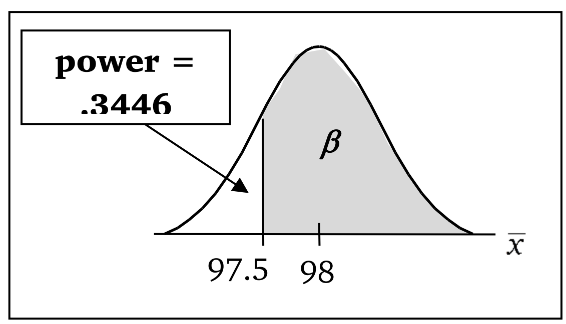
α	β
Reject H_0 if $\bar{x} < 97.5$	Don't reject H_0 if $\bar{x} \geq 97.5$
$\mu_0 = 100$	$\mu_a = 98$



The "β" column of our "α/β Table" has instructed us that β, the probability of Type II error, is the area to the right of 97.5, assuming the true mean is 98. Power is the unshaded region on this bell curve, the area to the left of 97.5 (since β and power add up to 1). Use the $\bar{x}$ standardizing formula to compute the z-score.

$$z = \frac{\bar{x} - \mu}{\sigma / \sqrt{n}} = \frac{97.5 - 98}{10 / \sqrt{64}} \rightarrow \boldsymbol{z = -0.4}$$

According to Table A, when $z = -0.4$, the left area is .3446. **The power = .3446**. It is sufficient to use the power to determine the reliability of this test. This is a very low power, so the test is not reliable.



Solution to Question 7 (f)

We have found the power is .3446 if the true mean is 98 hours. There is only a 34.46% chance that this experiment can be relied upon to spot the mean is <u>not</u> "at least 100 hours", as the company claims, if the true mean is 98 hours. This is a <u>not</u> a reliable test.

8. Bottles of a popular cola are supposed to contain 343 ml on average with a standard deviation of 3 ml. An inspector is concerned that the company is underfilling the bottles and, assuming the distribution is normal, plans to select 6 bottles at random to test this concern at the 5% level of significance. If the true mean fill is 342 ml:

(a) What is the power of this test?

(A) 0.2251 (B) 0.2033 (C) 0.7749 (D) 0.7967 (E) 0.8708

(b) The power of this test would increase if:

(A) The inspector would take more samples.

(B) The true mean was 340 ml.

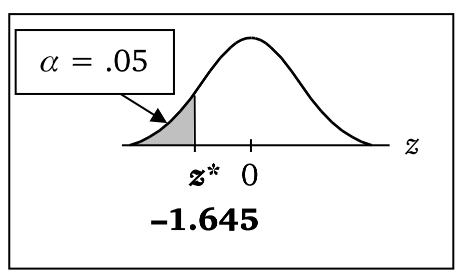
(C) The inspector used a higher level of significance.

(D) Only (A) and (C) are true.

(E) (A), (B) and (C) are all true.

<u>Givens</u>: $\boldsymbol{\mu = 343}$; $\boldsymbol{\sigma = 3}$ (we can use z). The company is suspected of **underfilling** the bottles ($\mu < 343$). The opposite statement is $\mu \geq 343$, so H_0 will take that one since it includes an equals sign, and H_a will take $\mu < 343$). Therefore, we can formulate the hypotheses $\boldsymbol{H_0: \mu = 343}$ ($\boldsymbol{\mu_0 = 343}$) vs. $\boldsymbol{H_a: \mu < 343}$ (a lower-tailed test). We are also told **the distribution is normal** (we can draw an $\bar{x}$-bell curve); $\boldsymbol{n = 6}$ (a very small sample, but that's OK since the population is assumed to be normal); $\boldsymbol{\alpha = 5\%}$; $\boldsymbol{\mu_a = 342}$.

If we are to compute the power, we need to determine the $\bar{x}$ decision rule and set up the "α/β Table" first.



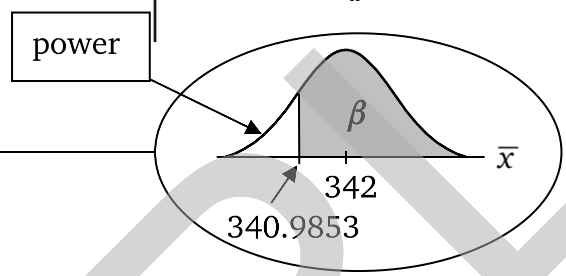
First, we need to know z^*, the critical z value. We were given $\alpha = 5\%$ and we have established we are doing a lower-tailed test, so visualize a z-bell curve with an area of .05 in its lower tail. Consulting Table D, we find $z^* = 1.645$ when the <u>upper</u> tail is ".05". BUT WE ARE DOING A LOWER-TAILED TEST. **Our critical value is $\boldsymbol{z^* = -1.645}$. (DO NOT FORGET TO INCLUDE THE "–" SIGN ON $\boldsymbol{z^*}$ WHEN YOUR TEST IS LOWER-TAILED!)**

$$\bar{x}^* = z^* \frac{\sigma}{\sqrt{n}} + \mu_0 = -1.645 \times \frac{3}{\sqrt{6}} + 343 \;\rightarrow\; \boldsymbol{\bar{x}^* = 340.9853}$$

So, our lower-tailed $\bar{x}$ decision rule is **reject H_0 if $\bar{x}$ < 340.9853**.

Now we are ready to make the "α/β Table" and compute the power.

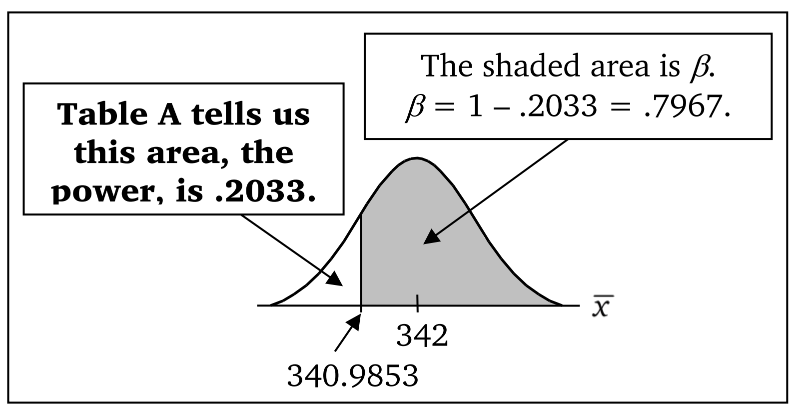
α	***β***
Reject H_0 if $\bar{x}$ < 340.9853	Don't reject H_0 if $\bar{x}$ ≥ 340.9853
$\mu_0 = 343$	$\mu_a = 342$



The "β" column of our "α/β Table" has instructed us that β, the probability of Type II error, is the area to the right of 340.9853, assuming the true mean is 342. However, this question has asked us to find power. Power is the unshaded region on this bell curve (since β and power add up to 1). **Thus, in this problem, the power is the left area on the bell curve!** Use the $\bar{x}$ standardizing formula to compute the z-score.

$$z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{340.9853 - 342}{3/\sqrt{6}} \;\rightarrow\; \boldsymbol{z = -0.83}$$

According to Table A, when $z = -0.83$, the left area is .2033. **Thus, the power is .2033.** (There is no need to get β first by doing 1 – .2033 = .7967 to get the right area, only to turn around and do 1 – β to get the power.)



Solution to Question 8 (a)

The power = .2033. The correct answer is (B).

8. (b) The power of this test would increase if:

(A) The inspector would take more samples.

(B) The true mean was 340 ml.

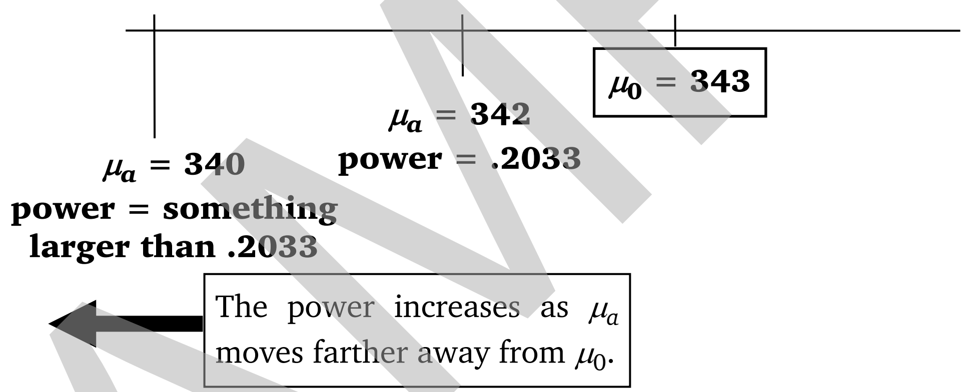
(C) The inspector used a higher level of significance.

(D) Only (A) and (C) are true.

(E) (A), (B) and (C) are all true.

(A) is TRUE. Recall the "chain reaction" that occurs if we increase the sample size. **If n ↑, then β ↓, power ↑.** Increasing the sample size will decrease the probability of Type II error, and so increase the power.

(B) is TRUE. They have given us a new alternative value for the mean, μ_a = **340**. We were given $\mu_0 = 343$ and $\mu_a = 342$ initially. Now, $\mu_a = 340$ is farther away from μ_0. **The farther away μ_a gets from μ_0, the larger the power gets.**



(C) is TRUE. A higher level of significance means a higher value for α. This, again, sets up a "chain reaction". **If α ↑, then β ↓, power ↑.** So, the power will increase if we increase α, the level of significance.

Solution to Question 8 (b)

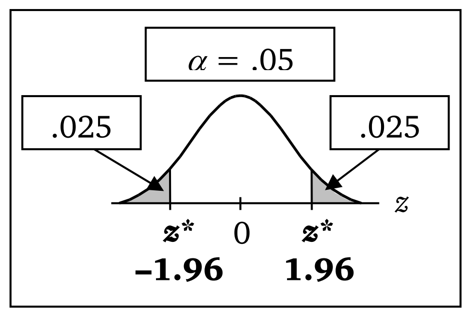
Thus, we know (A), (B) and (C) are all true.
The correct answer is (E).

9. A breakfast cereal claims to have an average of 2.7 grams of dietary fibre per 30 gram serving. A researcher, interested in seeing if the actual amount of fibre is different, assumes the population is normal with a standard deviation of 0.3 grams. Using a 5% significance level, she will measure the dietary fibre in 20 randomly selected 30-gram servings. If the true average is actually 2.5 grams, what is the probability of a Type II error?

(A) 0.8849 (B) 0.1539 (C) 0.8461 (D) 0.1151 (E) 0.8708

<u>Givens</u>: μ **= 2.7**; she is checking to see if "the amount of fibre is **different** ($\mu \neq$ **2.7**), so the hypotheses are $\boldsymbol{H_0}$**:** μ **= 2.7** (μ_0 **= 2.7**) vs. $\boldsymbol{H_a}$**:** $\mu \neq$ **2.7** (a two-tailed test); the population is **normal** (we can draw an $\bar{x}$-bell curve); σ **= 0.3** (we can use z); α **= 5%**; n **= 20**; μ_a **= 2.5**.

If we are to compute the probability of Type II error, we need to determine the $\bar{x}$ decision rule and set up the "α/β Table" first.



First, we need to know z^*, the critical z value. We were given α = 5% and we have established we are doing a two-tailed test, so visualize a z-bell curve with an area of .025 in each tail. Consulting Table D, we find z^* = 1.960 when the <u>upper</u> tail is ".025". But, we also have an equal and opposite lower tail at z^* = –1.960. Therefore, we have <u>two</u> critical values, $\boldsymbol{z^* = \pm 1.96}$. That means we need to compute both a lower and upper critical $\bar{x}$ value ($\bar{x}_L{}^*$ and $\bar{x}_U{}^*$).

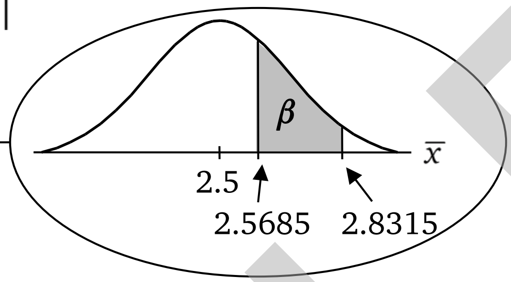
$$\boldsymbol{z^* = -1.96}: \quad \bar{x}_L{}^* = z^* \frac{\sigma}{\sqrt{n}} + \mu_0 = -1.96 \times \frac{0.3}{\sqrt{20}} + 2.7 \rightarrow \boldsymbol{\bar{x}_L{}^* = 2.5685}$$

$$\boldsymbol{z^* = 1.96}: \quad \bar{x}_U{}^* = z^* \frac{\sigma}{\sqrt{n}} + \mu_0 = 1.96 \times \frac{0.3}{\sqrt{20}} + 2.7 \rightarrow \boldsymbol{\bar{x}_U{}^* = 2.8315}$$

So, our two-tailed $\bar{x}$ decision rule is **reject $\boldsymbol{H_0}$ if $\boldsymbol{\bar{x}}$ < 2.5685 or $\boldsymbol{\bar{x}}$ > 2.8315**.

We are now ready to make the "α/β Table" and compute β.

α	β
Reject H_0 if $\bar{x} < 2.5685$ or $\bar{x} > 2.8315$	Don't reject H_0 if $2.5685 \le \bar{x} \le 2.8315$
$\mu_0 = 2.7$	$\mu_a = 2.5$

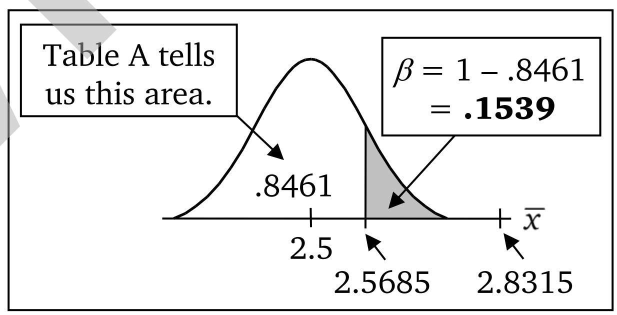


P*(Type II Error) = *β, and the "β" column of our "α/β Table" shows that β is the area between 2.5685 and 2.8315, assuming $\mu = 2.5$. Use the $\bar{x}$ standardizing formula to compute the two z-scores.

$$\bar{x} = \mathbf{2.5685}: \quad z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{2.5685 - 2.5}{0.3/\sqrt{20}} \rightarrow z = \mathbf{1.02}$$

$$\bar{x} = \mathbf{2.8315}: \quad z = \frac{\bar{x} - \mu}{\sigma/\sqrt{n}} = \frac{2.8315 - 2.5}{0.3/\sqrt{20}} \rightarrow z = \mathbf{4.94}$$

Of course, $z = 4.94$ is way off the end of Table A telling us 2.8315 is way off the end of the bell curve. Therefore, β, for all practical purposes, is simply the area to the right of 2.5685. When $z = 1.02$, the left area is .8461. The right area = ***P*(Type II Error) = *β* = 1 – .8461 = .1539**.



Solution to Question 9

The probability of Type II error is .1539.
The correct answer is (B).

SUMMARY OF KEY CONCEPTS IN LESSON 3

- Memorize the definitions of Type I and Type II errors.
  - **<u>TYPE I ERROR</u>: Rejecting H_0 when H_0 is true.**
  - **<u>TYPE II ERROR</u>: Not rejecting H_0 when H_0 is false.**
- The probability of Type I error is α, the level of significance. The probability of Type II error is β. There is <u>no</u> direct connection between α and β.
- If a researcher believes one error would have much more serious consequences than the other, they should set up their null and alternative hypotheses in such a way that the Type I error would be the more serious one. We have much more direct control over the probability of a Type I error since it equals α, the level of significance that we choose ourselves.
- **The <u>power of a test</u> is the *probability* we will *correctly* reject H_0.**
- **Power = 1 – β.** Put another way, power and β add up to 100%, so once we know one, we know the other.
- **The "α/β/Power" Chain**
  - **If $\alpha\downarrow$, then $\beta\uparrow$, power $\downarrow$.**
  - **If $\alpha\uparrow$, then $\beta\downarrow$, power $\uparrow$.**
- **The "*n*/β/Power" Chain**
  - **If $n\uparrow$, then $\beta\downarrow$, power $\uparrow$.**
  - **If $n\downarrow$, then $\beta\uparrow$, power $\downarrow$.**
- In order to compute α, β, or power for a hypothesis test for the mean, μ, we must first establish the $\bar{x}$ **decision rule**.
  - The formula to compute the critical $\bar{x}$ value is $\bar{x}^* = z^* \dfrac{\sigma}{\sqrt{n}} + \mu_0$.
  - Study the steps to determine the $\bar{x}$ decision rule on page 132.
- **We must be given an alternative mean, μ_a, in order to compute β or power** for a hypothesis test for the mean, μ.
- The easiest way to compute α, β, or power is to construct an **"α/β Table"**. Study the steps to construct an "α/β Table" on page 138.
- The farther μ_a gets from μ_0, in the appropriate direction, the higher the power will be.

LECTURE PROBLEMS FOR LESSON 3

For your convenience, here are the 9 questions I used as examples in this lesson. Do <u>not</u> make any marks or notes on these questions below. Especially, do not circle the correct choice in the multiple choice questions. You want to keep these questions untouched, so that you can look back at them without any hints. Instead, make any necessary notes, highlights, etc. in the lecture part above.

1. For the table below, indicate which are correct decisions and which are errors. If they are errors, what type?

		True Situation	
		H_0 is true	***H_0 is false***
Decision	***Reject H_0***		
	Not to reject H_0		

(See the solution on page 129.)

2. In testing a hypothesis you decide to lower the level of significance from 5% to 1%. Consider the following statements:

(I) The probability of Type I error has decreased from .05 to .01.
(II) The probability of Type II error has increased from .95 to .99.
(III) The power of the test has increased from .95 to .99.
(IV) The probability of Type II error has increased, but the amount cannot be determined.
(V) The probability of Type I error has decreased, but the amount cannot be determined.

(A) only **(I)** is true
(B) only **(I)** and **(II)** are true
(C) only **(I)** and **(III)** are true
(D) only **(I)** and **(IV)** are true
(E) only **(II)** and **(V)** are true

(See the solution on page 130.)

3. A researcher believes the average female executive at any company with more than 500 employees is making less than the 100 thousand dollars a year the average male executive makes. Consequently, she is going to test the hypotheses H_0: $\mu = 100$ thousand dollars vs. H_a: $\mu < 100$ thousand dollars. From previous research, it is known the distribution of executive salaries is approximately normal with $\sigma = 20$ thousand dollars. Determine the $\bar{x}$ decision rule in the following cases:

(a) She will select a random sample of 16 female executives and use a 10% level of significance.

(See the solution on page 134.)

(b) She will select a random sample of 16 female executives and use a 5% level of significance.

(See the solution on page 134.)

(c) She will select a random sample of 64 female executives and use a 5% level of significance.

(See the solution on page 135.)

(d) She will select a random sample of 64 female executives and use a 2% level of significance.

(See the solution on page 135.)

4. You have an SRS (simple random sample) of size $n = 16$ from a normal distribution with $\sigma = 6$. You wish to test H_0: $\mu = 12$ vs. H_a: $\mu > 12$, and you decide to reject H_0 if $\bar{x} > 15$.

(a) What is the probability of a Type I error?

(A) 0.0228 **(B)** 0.05 **(C)** 0.7486 **(D)** 0.4987 **(E)** 0.7734

(See the solution on page 141.)

(b) If $\mu = 16$ in fact, what is the probability of a Type II error?

(A) 0.0013 **(B)** 0.7266 **(C)** 0.7486 **(D)** 0.2734 **(E)** 0.2514

(See the solution on page 142.)

(c) If $\mu = 16$ in fact, what is the power of the test?

(A) 0.9987 **(B)** 0.7266 **(C)** 0.7486 **(D)** 0.2734 **(E)** 0.2514

(See the solution on page 142.)

5. You are examining a normal population with $\sigma = 10$. You wish to test H_0: $\mu = 30$ vs. H_a: $\mu \neq 30$ by taking a random sample of size 25. You decide to reject H_0 if $\bar{x} < 27$ or $\bar{x} > 33$.

(a) What is the probability of a Type I error?

(A) 0.0668 **(B)** 0.0548 **(C)** 0.1336 **(D)** 0.1096 **(E)** 0.05

(See the solution on page 144.)

(b) If $\mu = 28$ in fact, what is the power of the test?

(A) 0.3147 **(B)** 0.9332 **(C)** 0.05 **(D)** 0.6853 **(E)** 0.8664

(See the solution on page 145.)

(c) If $\mu = 25$ in fact, what is the power of the test?

(A) 0.1587 **(B)** 0.8849 **(C)** 0.8907 **(D)** 0.1093 **(E)** 0.8413

(See the solution on page 147.)

6. A sample of size 15 is selected from a normal population with variance 100. The null hypothesis H_0: $\mu = 77$ is tested against the alternative hypothesis H_a: $\mu > 77$ at the 5% level.

(a) What is the probability of a Type II error if μ is actually 83?

(A) 0.2517 **(B)** 0.7483 **(C)** 0.2483 **(D)** 0.7517 **(E)** 0.0500

(b) What is the probability of a Type I error?

(A) 0.2517 **(B)** 0.7483 **(C)** 0.2483 **(D)** 0.7517 **(E)** 0.0500

(See the solution on page 151.)

7. A researcher believes a company's claim its batteries last an average of at least 100 hours is bogus. He intends to take a random sample of 64 batteries and will reject their claim if $z < -2$. He assumes the population has a standard deviation of 10 hours.

(a) What is a Type I error in this context?

(See the solution on page 153.)

(b) What is a Type II error in this context?

(See the solution on page 153.)

(c) What level of significance is the researcher using?

(See the solution on page 155.)

(d) If the true mean is 90 hours, then what is the power of this test?

(See the solution on page 156.)

(e) If the true mean is 90 hours, is this a reliable test?

(See the solution on page 156.)

(f) If the true mean is 98 hours, is this a reliable test?

(See the solution on page 157.)

8. Bottles of a popular cola are supposed to contain 343 ml on average with a standard deviation of 3 ml. An inspector is concerned that the company is underfilling the bottles and, assuming the distribution is normal, plans to select 6 bottles at random to test this concern at the 5% level of significance. If the true mean fill is 342 ml:

(a) What is the power of this test?

(A) 0.2251 **(B)** 0.2033 **(C)** 0.7749 **(D)** 0.7967 **(E)** 0.8708

(See the solution on page 159.)

(b) The power of this test would increase if:

(A) The inspector would take more samples.
(B) The true mean was 340 ml.
(C) The inspector used a higher level of significance.
(D) Only (A) and (C) are true.
(E) (A), (B) and (C) are all true.

(See the solution on page 160.)

9. A breakfast cereal claims to have an average of 2.7 grams of dietary fibre per 30 gram serving. A researcher, interested in seeing if the actual amount of fibre is different, assumes the population is normal with a standard deviation of 0.3 grams. Using a 5% significance level, she will measure the dietary fibre in 20 randomly selected 30 gram servings. If the true average is actually 2.5 grams, what is the probability of a Type II error?

(A) 0.8849 **(B)** 0.1539 **(C)** 0.8461 **(D)** 0.1151 **(E)** 0.8708

(See the solution on page 162.)

HOMEWORK FOR LESSON 3

- Study the lesson thoroughly until you can do all of the Lecture Problems from start to finish without any assistance. **I have collected the Lecture Problems together for your convenience starting on page 164 above.**
- I have provided a **Summary of Key Concepts** starting on page 163 above.
- **Do not try to learn the material by doing your hand-in assignments. Learn the lesson first, then use the hand-in assignments to test your understanding of the lesson.** Before each hand-in assignment, I will send you tips telling you what lesson you should be studying to prepare for the assignment. Make sure you sign up for Grant's Homework Help at www.grantstutoring.com to receive these tips.
- If you have the *Multiple-Choice Problems Set for Basic Statistical Analysis II (Stat 2000)* by Smiley Cheng available in the Statistics section of the UM Book Store, then additional practise at many of the concepts taught in this lesson is available in:
    - **Section INM: Inference on Means. DO questions 1 to 100 ONLY at this time. You need knowledge of Lessons 4 and 5 before you can go further in this section. The solutions to this section are provided in Appendix B of my book starting on page B-1 below.** Please understand these solutions were written when they were using an earlier edition of the text. Specifically, you may see me referring to Table C when using t. That is what your text calls Table D. **Table C in my solutions is your Table D.**
- **Have you signed up for Grant's Homework Help yet?** Tips have already been sent to help you prepare for some of the hand-in assignments. Clear step-by-step instructions on how to get *JMP*™ to do the various things required have been sent as well. It's all FREE! Go to **www.grantstutoring.com** to sign up.
- **Grant's Midterm Exam Prep Seminar** has been scheduled by now. Go to **www.grantstutoring.com** to get all the info and register if you are interested.

APPENDIX A

HOW TO USE STAT MODES ON YOUR CALCULATOR

In the following pages, I show you how to enter data into your calculator in order to compute the mean and standard deviation. I also show you how to enter *x*, *y* data pairs in order to get the correlation, intercept and slope of the least squares regression line.

Please make sure that you are looking at the correct page when learning the steps. I give steps for several brands and models of calculator.

I consider it absolutely vital that a student know how to use the Stat modes on their calculator. It can considerably speed up certain questions and, even if a question insists you show all your work, gives you a quick way to check your answer.

If you cannot find steps for your calculator in this appendix, or cannot get the steps to work for you, do not hesitate to contact me. I am very happy to assist you in calculator usage (or anything else for that matter).

SHARP CALCULATORS
(Note that the EL-510 does not do Linear Regression.)

You will be using a "MODE" button. Look at your calculator. If you have "MODE" actually written on a button, press that when I tell you to press " MODE ". If you find mode written above a button (some models have mode written above the "DRG" button, like this: " DRG " with MODE above it) then you will have to use the " 2nd F " button to access the mode button; i.e. when I say " MODE " below, you will actually press " 2nd F DRG (MODE) ".

BASIC DATA PROBLEM

Feed in data to get the mean, $\bar{x}$, and standard deviation, *s* (which Sharps tend to denote "*sx*").

Step 1: Put yourself into the "STAT, SD" mode.

Press MODE 1 0 (Screen shows "Stat0")

Step 2: Enter the data: 3, 5, 9.
To enter each value, press the "M+" button. There are some newer models of Sharp that have you press the "CHANGE" button instead of the "M+" button. (The "CHANGE" button is found close by the "M+" button.)

3 M+ (DATA) 5 M+ (DATA) 9 M+ (DATA)

You should see the screen counting the data as it is entered (Data Set=1, Data Set=2, Data Set=3).

Step 3: Ask for the mean and standard deviation.

RCL 4 ($\bar{x}$)

We see that $\bar{x}$ =5.6666...=5.6667.

RCL 5 (sx)

We see that *s*=3.05505...=3.0551

Step 4: Return to "NORMAL" mode. This clears out your data as well as returning your calculator to normal.

MODE 0

LINEAR REGRESSION PROBLEM

Feed in *x* and *y* data to get the correlation coefficient, r, the intercept, a, and the slope, b.

Step 1: Put yourself into the "STAT, LINE" mode.

Press MODE 1 1 (Screen shows "Stat1")

Step 2: Enter the data:

x	3	5	9
y	7	10	14

Note you are entering in pairs of data (the *x* and *y* must be entered as a pair). The pattern is first *x*, press "STO" to get the comma, first *y*, then press "M+" (or **"CHANGE")** to enter the pair; repeat for each data pair.

3 STO ((x,y)) 7 M+ (DATA)

5 STO ((x,y)) 10 M+ (DATA)

9 STO ((x,y)) 14 M+ (DATA)

You should see the screen counting the data as it is entered (Data Set=1, Data Set=2, Data Set=3).

Step 3: Ask for the correlation coefficient, intercept, and slope. (The symbols may appear above different buttons than I indicate below.)

RCL ÷ (r)

We see that r=0.99419...=0.9942.

RCL ((a)

We see that a=3.85714...=3.8571.

RCL) (b)

We see that b=1.14285...=1.1429.

Step 4: Return to "NORMAL" mode. This clears out your data as well as returning your calculator to normal.

MODE 0

CASIO CALCULATORS
(Note that some Casios do not do Linear Regression.)

BASIC DATA PROBLEM

Feed in data to get the mean, $\bar{x}$, and standard deviation, *s* (which Casios tend to denote "$x\sigma_{n-1}$" or simply "σ_{n-1}").

Step 1: Put yourself into the "SD" mode.

Press "MODE" once or twice until you see "SD" on the screen menu and then select the number indicated. A little "SD" should then appear on your screen.

Step 2: Clear out old data.

SHIFT AC (Scl) = (Some models will have "Scl" above another button. Be sure you are pressing "Scl", the "Stats Clear" button. (Some models call it "SAC" for "Stats All Clear" instead of Scl.)

Step 3: Enter the data: 3, 5, 9.
To enter each value, press the "M+" button.

3 M+ (DT) 5 M+ (DT) 9 M+ (DT) (You use the "M+" button to enter each piece of data.)

Step 4: Ask for the mean and standard deviation.

SHIFT 1 ($\bar{x}$) =

We see that $\bar{x}$ = 5.6666...=5.6667.

SHIFT 3 ($x\sigma_{n-1}$) =

We see that *s* = 3.05505...=3.0551

(Some models may have $\bar{x}$ and $x\sigma_{n-1}$ above other buttons rather than "1" and "3" as I illustrate above.)

If you can't find these buttons on your calculator, look for a button called "S. VAR" (which stands for "Statistical Variables", it is probably above one of the number buttons).

Press: SHIFT S. VAR and you will be given a menu showing the mean and standard deviation. Select the appropriate number on the menu and press "=" (You may need to use your arrow buttons to locate the $\bar{x}$ or $x\sigma_{n-1}$ .options.)

Step 5: Return to "COMP" mode.

Press MODE and select the "COMP" option.

LINEAR REGRESSION PROBLEM

Feed in *x* and *y* data to get the correlation coefficient, r, the intercept, a, and the slope, b.

Step 1: Put yourself into the "REG, Lin" mode.

Press "MODE" once or twice until you see "Reg" on the screen menu and then select the number indicated. You will then be sent to another menu where you will select "Lin". (Some models call it the "LR" mode in which case you simply choose that instead.)

Step 2: Clear out old data.

Do the same as Step 2 for "Basic Data".

Step 3: Enter the data.

x	3	5	9
y	7	10	14

Note you are entering in pairs of data (the *x* and *y* must be entered as a pair). The pattern is first *x*, first *y*; second *x*, second *y*; and so on. Here is the data we want to enter:

3 , 7 M+ (DT) 5 , 10 M+ (DT) 9 , 14 M+ (DT)

(If you can't find the comma button " , ", you probably use the open bracket button instead to get the comma " [(-- ". You might notice "[x_D, y_D]" in blue below this button, confirming that is your comma.)

Step 4: Ask for the correlation coefficient, intercept, and slope. (The symbols may appear above different buttons than I indicate below.)

SHIFT ((r) =

We see that r = 0.99419...=0.9942.

SHIFT 7 (A) =

We see that a = 3.85714...=3.8571.

SHIFT 8 (B) =

We see that b = 1.14285...=1.1429.

If you can't find these buttons on your calculator, look for a button called "S. VAR"

Press: SHIFT S. VAR and you will be given a menu showing the mean and standard deviation. **Use your left and right arrow buttons to see other options**, like "*r*". Select the appropriate number on the menu and press "=".

Step 5: Return to "COMP" mode.

Press MODE and select the "COMP" option.

HEWLETT PACKARD HP 10B II

BASIC DATA PROBLEM

Feed in data to get the mean, $\bar{x}$, and standard deviation, *s* (which it denotes "*Sx*").

Step 1: Enter the data: 3, 5, 9.
To enter each value, press the "Σ+" button.

[3] [Σ+] [5] [Σ+] [9] [Σ+] (As you use the "Σ+" button to enter each piece of data, you will see the calculator count it going in: 1, 2, 3.)

Step 2: Ask for the mean and standard deviation.
Note that by "orange" I mean press the button that has the orange bar coloured on it. The orange bar is used to get anything coloured orange on the buttons.

[orange] [7 $\bar{x},\bar{y}$]

We see that $\bar{x}$ = 5.6666...=5.6667.

[orange] [8 s_x, s_y]

We see that *s* = 3.05505...=3.0551

Step 3: "Clear All" data ready for next time.

[orange] [C C ALL]

LINEAR REGRESSION PROBLEM

Feed in *x* and *y* data to get the correlation coefficient, r, the intercept, a, and the slope, b.

Step 1: Enter the data:

x	3	5	9
y	7	10	14

Note you are entering in pairs of data (the *x* and *y* must be entered as a pair). The pattern is first *x*, first *y*; second *x*, second *y*; and so on.

[3] [INPUT] [7] [Σ+]

[5] [INPUT] [10] [Σ+]

[9] [INPUT] [14] [Σ+]

(As you use the "Σ+" button to enter each pair of data, you will see the calculator count it going in: 1, 2, 3.)

Step 2: Ask for the correlation coefficient, intercept, and slope.

[orange] [4 $\hat{x},r$] [orange] [K SWAP]

We see that *r* = 0.99419...=0.9942.
Note that the "SWAP" button is used to get anything that is listed second (after the comma) like "*r*" in this case.

The intercept has to be found by finding $\hat{y}$ when *x*=0:

0 [orange] [5 $\hat{y},m$]

We see that *a* = 3.85714...=3.8571.
The slope is denoted "*m*" on this calculator:

[orange] [5 $\hat{y},m$] [orange] [K SWAP]

We see that *b* = 1.14285...=1.1429.

Step 3: "Clear All" data ready for next time.

[orange] [C C ALL]

TEXAS INSTRUMENTS TI-30X-II
(Note that the TI-30Xa does not do Linear Regression.)

BASIC DATA PROBLEM

Feed in data to get the mean, $\bar{x}$, and standard deviation, s (which it denotes "*Sx*").

Step 1: Clear old data.

[2nd] [DATA] (STAT) Use your arrow keys to ensure "*CLRDATA*" is underlined then press [ENTER =]

Step 2: Put yourself into the "STAT 1-Var" mode.

[2nd] [DATA] (STAT) Use your arrow keys to ensure "1-Var" is underlined then press [ENTER =]

Step 3: Enter the data: 3, 5, 9.

(You will enter the first piece of data as "X1", then use the down arrows to enter the second piece of data as "X2", and so on.)

[DATA] 3 [ENTER =] (X1 = 3)

[⇩] [⇩] 5 [ENTER =] (X2 = 5)

[⇩] [⇩] 9 [ENTER =] (X3 = 9)

Step 4: Ask for the mean and standard deviation.

Press [STATVAR] then you can see a list of outputs by merely pressing your left and right arrows to underline the various values.

We see that $\bar{x}$ = 5.6666...=5.6667.

We see that s = 3.05505...=3.0551

Step 5: Return to standard mode.

[CLEAR] This resets your calculator ready for new data next time.

LINEAR REGRESSION PROBLEM

Feed in *x* and *y* data to get the correlation coefficient, r, the intercept, a, and the slope, b.

Step 1: Clear old data (as in **BASIC DATA PROBLEM** at left).

Step 2: Put yourself into the "STAT 2-Var" mode.

[2nd] [DATA] (STAT) Use your arrow keys to ensure "2-Var" is underlined then press [ENTER =]

Step 3: Enter the data:

x	3	5	9
y	7	10	14

(You will enter the first *x*-value as "X1", then use the down arrow to enter the first *y*-value as "Y1", and so on.)

[DATA] 3 [ENTER =] [⇩] 7 [ENTER =] (X1 = 3, Y1 = 7)

[⇩] 5 [ENTER =] [⇩] 10 [ENTER =] (X2 = 5, Y2 = 10)

[⇩] 9 [ENTER =] [⇩] 14 [ENTER =] (X3 = 9, Y3 = 14)

Step 4: Ask for the correlation coefficient, intercept, and slope.

Press [STATVAR] then you can see a list of outputs by merely pressing your left and right arrows to underline the various values. **Note: Your calculator may have a and b reversed. To get a, you ask for b; to get b you ask for a.** Don't ask me why that is, but if that is the case then realize it will <u>always</u> be the case.

We see that r = 0.99419...=0.9942.

We see that a = 3.85714...=3.8571.

We see that b = 1.14285...=1.1429.

Step 5: Return to standard mode (as in **BASIC DATA PROBLEM** at left).

TEXAS INSTRUMENTS TI-36X
(Note that the TI-30Xa does not do Linear Regression.)

BASIC DATA PROBLEM

Feed in data to get the mean, $\bar{x}$, and standard deviation, *s* (which it denotes "σx_{n-1}").

Step 1: Put yourself into the "STAT 1" mode.

[3rd] [STAT 1 / x ⇄ y]

Step 2: Enter the data: 3, 5, 9.
To enter each value, press the "Σ+" button.

3 [Σ+] 5 [Σ+] 9 [Σ+] (As you use the "Σ+" button to enter each piece of data, you will see the calculator count it going in: 1, 2, 3.)

Step 3: Ask for the mean and standard deviation.

[2nd] [$\bar{x}$ / x^2]

We see that $\bar{x}$ = 5.6666...=5.6667.

[2nd] [σ_{xn-1} / $\sqrt{x}$]

We see that *s* = 3.05505...=3.0551

Step 4: Return to standard mode.

[ON/AC] (Be careful! If you ever press this button during your work you will end up resetting your calculator and losing all of your data. Use the [CE/C] button to clear mistakes without resetting your calculator. I usually press this button a couple of times to make sure it has cleared any mistake completely.)

LINEAR REGRESSION PROBLEM

Feed in *x* and *y* data to get the correlation coefficient, r, the intercept, a, and the slope, b.

Step 1: Put yourself into the "STAT 2" mode.

[3rd] [STAT 2 / Σ+]

Step 2: Enter the data:

x	3	5	9
y	7	10	14

Note you are entering in pairs of data (the *x* and *y* must be entered as a pair). The pattern is first *x*, first *y*; second *x*, second *y*; and so on.

3 [$x \rightleftarrows y$] 7 [Σ+]

5 [$x \rightleftarrows y$] 10 [Σ+]

9 [$x \rightleftarrows y$] 14 [Σ+]

(As you use the "Σ+" button to enter each pair of data, you will see the calculator count it going in: 1, 2, 3.)

Step 3: Ask for the correlation coefficient, intercept, and slope.

Note that this calculator uses the abbreviations "COR" for correlation, "ITC" for intercept and "SLP" for slope.

[3rd] [COR / 4]

We see that r = 0.99419...=0.9942.

[2nd] [ITC / 4]

We see that a = 3.85714...=3.8571.

[2nd] [SLP / 5]

We see that b = 1.14285...=1.1429.

Step 4: Return to standard mode.

[ON/AC]

TEXAS INSTRUMENTS TI-BA II Plus

Put yourself into the "LIN" mode.

[2nd] [8] (STAT) If "LIN" appears, great; if not, press [2nd] [ENTER] (SET) repeatedly until "LIN" does show up. Then press [2nd] [CPT] (QUIT) to "quit" this screen.

Note: Once you have set the calculator up in "LIN" mode, it will stay in that mode forever. You can now do either "Basic Data" or "Linear Regression" problems.

BASIC DATA PROBLEM

Feed in data to get the mean, $\bar{x}$, and standard deviation, *s* (which it denotes "*Sx*").

Step 1: Clear old data.

[2nd] [7] (DATA) [2nd] [CE/C] (CLR Work)

Step 2: Enter the data: 3, 5, 9.

(You will enter the first piece of data as "X1", then use the down arrows to enter the second piece of data as "X2", and so on. Ignore the "Y1", "Y2", etc.)

[DATA] 3 [ENTER =] $(X1 = 3)$

[⇓] [⇓] 5 [ENTER =] $(X2 = 5)$

[⇓] [⇓] 9 [ENTER =] $(X3 = 9)$

Step 3: Ask for the mean and standard deviation.

Press [2nd] [8] (STAT) then you can see a list of outputs by merely pressing your up and down arrows to reveal the various values.

We see that $\bar{x}$ = 5.6666...=5.6667.

We see that *s* = 3.05505...=3.0551

Step 4: Return to standard mode.

[ON/OFF] This resets your calculator ready for new data next time.

LINEAR REGRESSION PROBLEM

Feed in *x* and *y* data to get the correlation coefficient, r, the intercept, a, and the slope, b.

Step 1: Clear old data.

[2nd] [7] (DATA) [2nd] [CE/C] (CLR Work)

Step 2: Enter the data:

x	3	5	9
y	7	10	14

(You will enter the first *x*-value as "X1", then use the down arrow to enter the first *y*-value as "Y1", and so on.)

[DATA] 3 [ENTER =] [⇓] 7 [ENTER =] $(X1 = 3, Y1 = 7)$

[⇓] 5 [ENTER =] [⇓] 10 [ENTER =] $(X2 = 5, Y2 = 10)$

[⇓] 9 [ENTER =] [⇓] 14 [ENTER =] $(X3 = 9, Y3 = 14)$

Step 3: Ask for the correlation coefficient, intercept, and slope.

Press [2nd] [8] (STAT) then you can see a list of outputs by merely pressing your up and down arrows to reveal the various values.
We see that r = 0.99419...=0.9942.

We see that a = 3.85714...=3.8571.

We see that b = 1.14285...=1.1429.

Step 4: Return to standard mode.

[ON/OFF] This resets your calculator ready for new data next time.