

eg. $T: \mathbb{R}^3 \rightarrow \mathbb{R}^3$

$$T(\vec{e}_1, -\vec{e}_2) = (1, 3, 7)$$

$$T(\vec{e}_1 + \vec{e}_3) = (1, 5, 10)$$

$$T(\vec{e}_2) = (0, 0, 4)$$

(a) Find $T(\vec{e}_1)$

(b) Find $T(\vec{e}_3)$

(c) Find the standard matrix of the transformation.

Answer

(a) We know $T(\vec{e}_1 - \vec{e}_2) = (1, 3, 7)$

$$\therefore T(\vec{e}_1) - T(\vec{e}_2) = (1, 3, 7)$$

(using properties of Linear Transformation)

We were given $T(\vec{e}_2) = (0, 0, 4)$

$$\therefore T(\vec{e}_1) - (0, 0, 4) = (1, 3, 7)$$

$$T(\vec{e}_1) = (1, 3, 7) + (0, 0, 4)$$

$$\boxed{T(\vec{e}_1) = (1, 3, 11)}$$

Answers part (a)

(b) Given $T(\vec{e}_1 + \vec{e}_3) = (1, 5, 10)$

$$T(\vec{e}_1) + T(\vec{e}_3) = (1, 5, 10)$$

We solved $T(\vec{e}_1) = (1, 3, 11)$

$$(1, 3, 11) + T(\vec{e}_3) = (1, 5, 10)$$

$$T(\vec{e}_3) = (1, 5, 10) - (1, 3, 11)$$

$$\boxed{T(\vec{e}_3) = (0, 2, -1)}$$

answers part (b)

(c) We now know
 $T(\vec{e}_1) = (1, 3, 11) \rightarrow$ found in (a)

$T(\vec{e}_2) = (0, 0, 4) \rightarrow$ given

$T(\vec{e}_3) = (0, 2, -1) \rightarrow$ found in (b)

standard matrix "A" or "[T]"

is $A = \begin{bmatrix} 1 & 0 & 0 \\ 3 & 0 & 2 \\ 11 & 4 & -1 \end{bmatrix}$