

Given $T: \mathbb{R}^2 \rightarrow \mathbb{R}^2$

such that $T(\vec{e}_1 + 2\vec{e}_2) = (4, 5)$

and $T(\vec{e}_1 - 3\vec{e}_2) = (7, 8)$

(a) Find $T(\vec{e}_2)$

(b) Find $T(\vec{e}_1)$

(c) Write the standard matrix
A for the transformation

Answers

(a) Given $T(\vec{e}_1 + 2\vec{e}_2) = (4, 5)$
and $T(\vec{e}_1 - 3\vec{e}_2) = (7, 8)$

Therefore: $T(\vec{e}_1) + 2T(\vec{e}_2) = (4, 5)$
 $T(\vec{e}_1) - 3T(\vec{e}_2) = (7, 8)$

Subtract "Top - Bottom"

creates

$T(\vec{e}_1)$	$+ 2T(\vec{e}_2)$	$(4, 5)$
$- T(\vec{e}_1)$	$- (-3)T(\vec{e}_2)$	$-(7, 8)$
<u>Gone</u>	<u>$+ 5T(\vec{e}_2)$</u>	<u>$(-3, -3)$</u>

$$T(\vec{e}_2) = \frac{(-3, -3)}{5}$$

$$T(\vec{e}_2) = \left(-\frac{3}{5}, -\frac{3}{5}\right)$$

Answers part (a)

(b) Now that we know

$$T(\vec{e}_2) = \left(-\frac{3}{5}, -\frac{3}{5}\right)$$

Use $T(\vec{e}_1 + 2\vec{e}_2) = (4, 5)$

$$T(\vec{e}_1) + 2T(\vec{e}_2) = (4, 5)$$

$$T(\vec{e}_1) + 2\left(-\frac{3}{5}, -\frac{3}{5}\right) = (4, 5)$$

$$T(\vec{e}_1) = (4, 5) - \left(-\frac{6}{5}, -\frac{6}{5}\right)$$

$$T(\vec{e}_1) = \left(4 + \frac{6}{5}, 5 + \frac{6}{5}\right)$$

$$T(\vec{e}_1) = \left(\frac{26}{5}, \frac{31}{5}\right)$$

Answers part (b)

(c)

$$A = \begin{pmatrix} \frac{26}{5} & -\frac{3}{5} \\ \frac{31}{5} & -\frac{3}{5} \end{pmatrix}$$