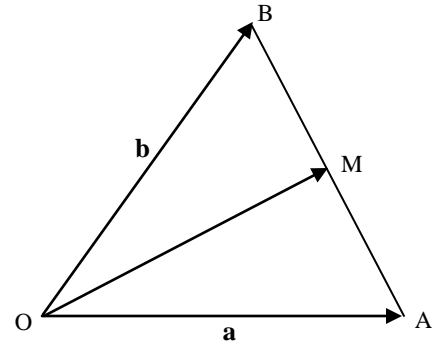


## MATH 1300 ASSIGNMENT PROBLEMS (UNIT 1)

[10] 1. OAB is an isosceles triangle with  $OA = OB$  and M is the mid-point of AB. Let  $\overrightarrow{OA} = \mathbf{a}$  and let  $\overrightarrow{OB} = \mathbf{b}$ .

(a) Write the vectors  $\overrightarrow{AB}$  and  $\overrightarrow{OM}$  as linear combinations of the vectors  $\mathbf{a}$  and  $\mathbf{b}$ .

(b) Use vector methods to show that  $\overrightarrow{OM}$  is perpendicular to  $\overrightarrow{AB}$ .



[10] 2. Let  $\mathbf{u} = (1, -3, 2)$ ,  $\mathbf{v} = (3, 1, -2)$  and  $\mathbf{w} = (4, 0, 1)$  be three vectors in  $\mathbf{R}^3$ . Find the following.

(a)  $3\mathbf{u} - 2\mathbf{v} + 4\mathbf{w}$

(b)  $\mathbf{u} \cdot \mathbf{w}$

(c)  $\mathbf{v} \times \mathbf{w}$

(d)  $\text{proj}_{\mathbf{v}} \mathbf{u}$

(e) cosine of the angle between the vectors  $\mathbf{u}$  and  $\mathbf{v}$ .

[6] 3. Show that the 3 points  $P=(1, 3, 4)$ ,  $Q=(3, 2, 5)$  and  $R=(5, 1, 6)$  all lie on the same straight line.

[4] 4. Let  $\mathbf{u} = (3, 1, 0)$  and  $\mathbf{v} = (1, 2, c)$  be two vectors in  $\mathbf{R}^3$ . For what value(s) of  $c$  is the angle between the vectors equal to  $60^\circ$ ?

[10] 5. Let  $\mathbf{u} = (8, 12, 1)$  and  $\mathbf{v} = (4, 6, k)$  be two vectors in  $\mathbf{R}^3$ .

(a) For what value(s) of  $k$  will the two vectors  $\mathbf{u}$  and  $\mathbf{v}$  be parallel? Explain.

(b) For what value(s) of  $k$  will the two vectors  $\mathbf{u}$  and  $\mathbf{v}$  be orthogonal? Explain.

(c) For what value(s) of  $k$  will the two vectors  $\mathbf{u}$  and  $\mathbf{v}$  be of equal length?

[10] 6. Let  $l: 2x + 5y = 9$  be a line and  $P = (3, 6)$  be a point in  $\mathbf{R}^2$ .

(a) Let  $Q$  be the point on the line  $l$  having its  $y$ -coordinate = 0 and let  $R$  be the point on the line  $l$  having its  $x$ -coordinate = 0. Find the coordinates of the points  $Q$  and  $R$ .

(b) Plot the points  $P, Q, R$  and the line  $l$  on a two-dimensional Cartesian coordinate system.

(c) Find the components of the vector  $\overrightarrow{QP}$ .

(d) Find a normal vector  $\mathbf{n}$  to the given line  $l$ .

(e) Find the distance between the point  $P$  and the line  $l$ .

[10] 7. The plane  $x + 2y + 2z = 4$  intersects the positive coordinate axis  $OX, OY$  and  $OZ$  in three points  $A, B$  and  $C$  respectively.

(a) Find the coordinates of the three points  $A, B$  and  $C$ .

(b) Find the area of the triangle  $ABC$ .

