MATH 1300 ASSIGNMENT PROBLEMS (UNIT 2)

[10] 1. Let P = (2, 3, 1), Q = (4, 1, 2) and R = (1, 2,-3) be 3 points in $\mathbf{R}^3$.

(a) Find the components of the vector $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

(b) Find a set of parametric equations for the line through the points P and R.

(c) Use the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ to find a normal vector to the plane through the 3 points P, Q and R.

(d) Find a standard form equation of the plane through the 3 points P, Q and R.

[10] 2. A plane passes through 3 non-collinear points P = (–2, 1, 0), Q = (–1, 3, 1) and R = (2, –2, 3).

(a) Find the standard form equation of the plane.
(b) The line l passes the points P and Q. Find its two-point vector form equation. Find the corresponding parametric equations.

[10] 3. Let $l: (x, y, z) = (4, 2, 3) + t(5, 3, -2)$ be a line in $\mathbf{R}^3$ and let $\boldsymbol{\pi}: x - y + z = 4$ be a plane in $\mathbf{R}^3$.

(a) Find a vector $\mathbf{v}$ parallel to line l and a vector $\mathbf{n}$ that is normal to the plane $\boldsymbol{\pi}$.

(b) Show that the line l is parallel to the plane $\boldsymbol{\pi}$.

(c) Set $t = 0$ in the vector equation for the line l to find a point P on the line l. Set $y = z = 0$ to find a point Q on the plane $\boldsymbol{\pi}$. Find the components of the vector $\overrightarrow{QP}$.

(d) Find the distance between the line l and the plane $\boldsymbol{\pi}$.

(e) Find the point of intersection of the line $(x, y, z) = (2, 3, -1) + t(3, -1, 2)$ and the plane $\boldsymbol{\pi}$.

[10] 4. Given the skew lines $l_1:\ x=2+3t,\ \ y=3-t,\ \ z=1+2t$ and
$l_2:\ x=3-2s,\ \ y=5+3s,\ \ z=1+s$, find the following.

(a) A vector $\mathbf{v_1}$ parallel to line l_1 and a vector $\mathbf{v_2}$ parallel to line l_2.

(b) A vector **n** that is orthogonal to both lines l_1 and l_2.

(c) Sine of the angle between the lines l_1 and l_2.

(d) A point P on line l_1 and a point Q on line l_2. Find also the vector $\overrightarrow{\mathrm{PQ}}$.

(e) The distance between the lines l_1 and l_2.

[10] 5. Given the point P = (3, 1, 2) and the plane $2x - 3y + z = 1$, find the following.

(a) A set of parametric equations for the line through P that is also orthogonal to the given plane.

(b) The point of intersection of the line from part (a) with the given plane.

[10] 6. Given $\boldsymbol{\pi}_1:\ x+ay+3z=6$ and $\boldsymbol{\pi}_2:\ ax+4y+6z=4$ are standard form equations of two planes in $\mathbf{R^3}$, find the following.

(a) A normal vector $\mathbf{n_1}$ to the plane $\boldsymbol{\pi}_1$ and a normal vector $\mathbf{n_2}$ to the plane $\boldsymbol{\pi}_2$.

(b) For what value(s) of a are these two planes parallel to each other?

(c) For what value(s) of a are these two planes perpendicular to each other?

(d) If $a = 1$, these two planes intersect each other. Find the cosine of the dihedral angle between the two planes when $a = 1$.

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