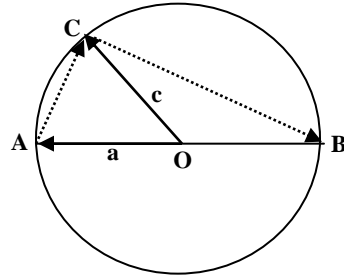


## MATH 1300 ASSIGNMENT PROBLEMS (UNIT 1)

- [10] 1.  $AOB$  is the diameter of a circle with centre at  $O$  and  $C$  is any other point on the circle. Denote the vector  $\overrightarrow{OA}$  by  $\mathbf{a}$  and the vector  $\overrightarrow{OC}$  by  $\mathbf{c}$ .



- (a) Write the vectors  $\overrightarrow{AC}$  and  $\overrightarrow{CB}$  as linear combinations of the vectors  $\mathbf{a}$  and  $\mathbf{c}$ .
- (b) Use vector methods to show that  $\angle ACB$  is a right angle.
- [10] 2. Let  $A = (2, 1, -5)$  and  $B = (1, -2, 4)$  be two points in  $\mathbf{R}^3$ .
- (a) Find the components of the vectors  $\overrightarrow{AB}$  and  $\overrightarrow{BA}$ .
- (b) Find the coordinates of the point  $C$  if  $\overrightarrow{AC} = \overrightarrow{CB}$ .
- (c) The point  $D = (k, 1, 5)$  is equidistant from the points  $A$  and  $B$ . Find the value(s) of  $k$ .
- (d) Find the coordinates of the point  $X$  for which  $\overrightarrow{AX} = 2\overrightarrow{AB}$ .
- [10] 3. Let  $\mathbf{u} = (4, 3, 1)$ ,  $\mathbf{v} = (5, 4, -2)$  and  $\mathbf{w} = (7, -3, 4)$  be three vectors in  $\mathbf{R}^3$ . Find the following.
- (a)  $2\mathbf{u} - 3\mathbf{v} + \mathbf{w}$
- (b)  $\mathbf{u} \cdot \mathbf{v}$
- (c)  $\mathbf{v} \times \mathbf{u}$
- (d)  $\text{proj}_{\mathbf{v}} \mathbf{u}$
- (e) sine of the angle between the vectors  $\mathbf{u}$  and  $\mathbf{v}$ .

- [10] 4. Let  $\mathbf{u} = (4, 2, 7)$  and  $\mathbf{v} = (2, 1, k)$  be two vectors in  $\mathbf{R}^3$ .
- For what value(s) of  $k$  will the two vectors  $\mathbf{u}$  and  $\mathbf{v}$  be parallel? Explain.
  - For what value(s) of  $k$  will the two vectors  $\mathbf{u}$  and  $\mathbf{v}$  be orthogonal? Explain.
  - For what value(s) of  $k$  will the two vectors  $\mathbf{u}$  and  $\mathbf{v}$  be of equal length?

- [10] 5. Let  $l: 3x + 4y = 12$  be a line and  $P = (6, 7)$  be a point in  $\mathbf{R}^2$ .
- Let  $Q$  be the point on the line  $l$  having its  $y$ -coordinate = 0 and let  $R$  be the point on the line  $l$  having its  $x$ -coordinate = 0. Find the coordinates of the points  $Q$  and  $R$ .
  - Plot the points  $P, Q, R$  and the line  $l$  on a two-dimensional Cartesian coordinate system.
  - Find the components of the vector  $\overrightarrow{QP}$ .
  - Find a normal vector  $\mathbf{n}$  to the given line  $l$ .
  - Find the distance between the point  $P$  and the line  $l$ .

- [10] 6. The four points  $A(5,0,0)$ ,  $B(0,0,2)$ ,  $C(0,4,0)$  and  $D(5,6,-3)$  form a quadrilateral lying on the plane  $4x + 5y + 10z = 20$ . Find the area of this quadrilateral.

