MATH 1300 ASSIGNMENT PROBLEMS (UNIT 2)

[10] 1. Let P = (1, 3, –1), Q = (2, 1, –2) and R = (–2, 1,–3) be 3 points in $\mathbf{R}^3$.

(a) Find the components of the vector $\overrightarrow{PQ}$ and $\overrightarrow{PR}$.

(b) Find a set of parametric equations for the line through the points P and R.

(c) Use the vectors $\overrightarrow{PQ}$ and $\overrightarrow{PR}$ to find a normal vector to the plane through the 3 points P, Q and R.

(d) Find a standard form equation of the plane through the 3 points P, Q and R.

[10] 2. Let $\boldsymbol{\pi}_1 : 3x-2y+2z=6$ and $\boldsymbol{\pi}_2 : 2x-3y+z=9$ be two planes in $\mathbf{R}^3$.

(a) Find a normal vector $\mathbf{n_1}$ to the plane $\boldsymbol{\pi}_1$ and a normal vector $\mathbf{n_2}$ to the plane $\boldsymbol{\pi}_2$.

(b) Find the cosine of the dihedral angle between the planes $\boldsymbol{\pi}_1$ and $\boldsymbol{\pi}_2$.

(c) Find a vector **v** parallel to the line of intersection of the planes $\boldsymbol{\pi}_1$ and $\boldsymbol{\pi}_2$.

(d) Find the point on the line of intersection of the planes $\boldsymbol{\pi}_1$ and $\boldsymbol{\pi}_2$ whose y-coordinate is 0.

(e) Use the results from parts (c) and (d) to find a set of parametric equations of the line of intersection of the planes $\boldsymbol{\pi}_1$ and $\boldsymbol{\pi}_2$.

[10] 3. Let $l : (x, y, z) = (-1, 2, 2) + t(3, 1, -1)$ be a line in $\mathbf{R}^3$ and let $\boldsymbol{\pi} : x-2y+z=5$ be a plane in $\mathbf{R}^3$.

(a) Find a vector **v** parallel to line l and a vector **n** that is normal to the plane $\boldsymbol{\pi}$.

(b) Show that the line l is parallel to the plane $\boldsymbol{\pi}$.

(c) Set $t = 0$ in the vector equation for the line l to find a point P on the line l. Set $y = z = 0$ to find a point Q on the plane $\boldsymbol{\pi}$. Find the components of the vector $\overrightarrow{QP}$.

(d) Find the distance between the line l and the plane $\boldsymbol{\pi}$.

(e) Find the point of intersection of the line $(x, y, z) = (2, 3, -1) + t(1, 2, -1)$ and the plane.

[10] 4. Given the skew lines $l_1: \ x = 1 - 2t, \ y = 3 + t, \ z = 1 + 3t$ and $l_2: \ x = 1 + s, \ y = 2 - 2s, \ z = -1 + 3s$, find the following.

(a) A vector $\mathbf{v_1}$ parallel to line l_1 and a vector $\mathbf{v_2}$ parallel to line l_2.

(b) A vector **n** that is orthogonal to both lines l_1 and l_2.

(c) Sine of the angle between the lines l_1 and l_2.

(d) A point P on line l_1 and a point Q on line l_2. Find also the vector $\overrightarrow{PQ}$.

(e) The distance between the lines l_1 and l_2.

[10] 5. Given the point P = (2, 1, 3) and the plane $x - 2y + 2z = 7$, find the following.

(a) A set of parametric equations for the line through P that is also orthogonal to the given plane.

(b) The point of intersection of the line from part (a) with the given plane.

[10] 6. Given $\boldsymbol{\pi}_1: \ x + ay + 2z = 5$ and $\boldsymbol{\pi}_2: \ ax + 9y + 6z = 12$ are standard form equations of two planes in $\mathbf{R^3}$, find the following.

(a) A normal vector $\mathbf{n_1}$ to the plane $\boldsymbol{\pi}_1$ and a normal vector $\mathbf{n_2}$ to the plane $\boldsymbol{\pi}_2$.

(b) For what value(s) of *a* are these two planes parallel to each other?

(c) For what value(s) of *a* are these two planes perpendicular to each other?

(d) If $a = 1$, these two planes intersect each other. Find the cosine of the dihedral angle between the two planes when $a = 1$.