MATH 1300 ASSIGNMENT PROBLEMS (UNIT 4)

[10] 1. Let $A = \begin{bmatrix} 2 & -1 & 3 \\ 4 & 0 & 1 \\ 3 & 5 & -2 \end{bmatrix}$ and let $B = \begin{bmatrix} 5 & 3 & -1 \\ 2 & 4 & 3 \\ 1 & -2 & 0 \end{bmatrix}$. Find the following.

(a) $A+2B^T$

(b) AB

(c) BA

(d) The matrix C for which $2A + C^T = B$.

[10] 2.(a) Which of the following matrices are elementary matrices?

(i) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(b) Let $A = \begin{bmatrix} 1 & 4 & 3 \\ 2 & 1 & 5 \\ 3 & 2 & 4 \end{bmatrix}$. Find an elementary matrix E such that EA = B if

(i) $B = \begin{bmatrix} 1 & 4 & 3 \\ 4 & 9 & 11 \\ 3 & 2 & 4 \end{bmatrix}$ (ii) $B = \begin{bmatrix} 3 & 2 & 4 \\ 2 & 1 & 5 \\ 1 & 4 & 3 \end{bmatrix}$ (iii) $B = \begin{bmatrix} 1 & 4 & 3 \\ 2 & 1 & 5 \\ 9 & 6 & 12 \end{bmatrix}$

(c) Let $A = \begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$. Find two elementary matrices E_1 and E_2 such that $E_2 E_1 A = I$.

[10] 3. Find the inverse of $A = \begin{bmatrix} 1 & 1 & 1 & 0 \\ 2 & 3 & 3 & 1 \\ 3 & 3 & 4 & 3 \\ 4 & 4 & 4 & 1 \end{bmatrix}$. Show all your work and verify that your answer is correct.

[10] 4. Consider the following system of linear equations

$$\begin{aligned} x_1 + x_2 + x_3 \quad &= 4 \\ 2x_1 + 3x_2 + 3x_3 + 1x_4 &= 3 \\ 3x_1 + 3x_2 + 4x_3 + 3x_4 &= 2 \\ 4x_1 + 4x_2 + 4x_3 + 1x_4 &= 1 \end{aligned}.$$

(a) Rewrite this system of linear equations as a single matrix equation in the form $A\mathbf{x} = \mathbf{b}$.

(b) Use the matrix A^{-1} to find the solution **x**. [Hint: See problem 3 for A^{-1}.]

[10] 5. Provide examples to illustrate the following.

(a) A 3×3 matrix B such that $B^2 = I$ with $B \neq I$ and $B \neq -I$.

(b) A 3×3 nonzero matrix C such that $C^3 = O$ but $C^2 \neq O$.

(c) A 3×3 matrix D such that $D^T = D$ with $D \neq I$ and $D \neq O$.

(d) A 3×3 matrix F such that $F^T = -F$ with $F \neq O$.

[10] 6. The citizens of Oz have a choice of 3 political parties in their municipal elections, the Blue party, the Green party or the Red party. A study of past voting patterns shows that if a citizen voted for the Blue party in one election, the probability that he/she will vote for the Blue party in the next election is 70%, the probability he/she will vote for the Green party is 20% and the probability he/she will vote for the Red party is 10%. If a citizen voted for the Green party in one election, the probability that he/she will vote for Green party in the next election is 60%, the probability he/she will vote for the Blue party is 20% and the probability that he/she will vote for the Red party is 20%. If a citizen voted for the Red party in one election, the probability that he/she will vote for the Red party in the next election is 50%, the probability that he/she will vote for the Blue party is 30% and the probability that he/she will vote for the Green party is 20%.

(a) Find the transition matrix for the voting intentions of the citizens of Oz.

(b) If the vote distribution at the last election was Blue 50%, Green 30%, Red 20%, find the probable vote distribution at the next election.

(c) Find the long term steady state distribution of the votes in Oz.