

MATH 1300 ASSIGNMENT PROBLEMS (UNIT 3)

- [10] 1. Determine into which of the following 3 types (A), (B) or (C) the matrices (a) to (e) below can be classified.

Type (A): The matrix is in both reduced row-echelon form and row-echelon form.

Type (B): The matrix is in row-echelon form but not in reduced row-echelon form.

Type (C): The matrix is in neither reduced row-echelon form nor in row-echelon form.

$$(a) \begin{bmatrix} 1 & 0 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (b) \begin{bmatrix} 1 & 1 & 0 & 0 \\ 0 & 0 & 1 & 1 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (c) \begin{bmatrix} 1 & 0 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 0 & 1 \end{bmatrix} \quad (d) \begin{bmatrix} 1 & 2 & 0 & 3 \\ 0 & 0 & 1 & 4 \\ 0 & 0 & 0 & 1 \\ 0 & 0 & 0 & 0 \end{bmatrix} \quad (e) \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix}$$

(f) Find all 3×2 reduced row-echelon matrices. Note: This question is independent of parts (a-e). Hint: There are 4 such matrices.

- [10] 2. Consider the following system of linear equations.

$$x_1 + x_3 + x_4 = 5$$

$$x_1 + x_2 - x_3 = 2$$

$$x_2 + x_3 + x_4 = 3$$

$$x_1 - x_2 + x_4 = 2$$

- (a) Write out the augmented matrix for this system of linear equations.
- (b) Use elementary row operations to reduce the augmented matrix to reduced row-echelon form.
- (c) Write out the solution to the system of linear equations.

- [10] 3. The augmented matrix from a system of linear equations has the following reduced row-echelon form.

$$\begin{bmatrix} 1 & 3 & 0 & 5 & 0 & 4 & 0 & 2 \\ 0 & 0 & 1 & 2 & 0 & 2 & 0 & 3 \\ 0 & 0 & 0 & 0 & 1 & 3 & 0 & 4 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 5 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 & 0 \end{bmatrix}$$

- (a) How many equations are there in the system?
- (b) How many variables are there in the system?
- (c) How many of the variables are independent variables?
- (d) Write out the solution set for the system.

[10] 4. Consider the system of linear equations

$$\begin{aligned}x + 3y &= 5 \\ 2x + ay &= b\end{aligned}$$

where a and b are real numbers.

- (a) Write out the augmented matrix for this system of linear equations.
- (b) Use elementary row operations to reduce the augmented matrix to row-echelon form.
- (c) Determine for what values of a and b does the system have infinitely many solutions.
- (d) Determine for what values of a and b does the system have no solution.
- (e) Determine for what values of a and b does the system have an unique solution.

[10] 5. Anne, Betty and Carol went to their local produce store to buy some fruit. Anne bought one pound of apples and two pounds of bananas and paid \$1.98. Betty bought two pounds of apples and one pound of grapes and paid \$3.45. Carol bought one pound of bananas and two pounds of grapes and paid \$3.43.

- (a) Let x = price of a pound of apples, y = price of a pound of bananas and z = price of a pound of grapes. Write out 3 linear equations representing the purchases of Anne, Betty and Carol.
- (b) Write out the augmented matrix for your system of 3 linear equations of part (a).
- (c) Use elementary row operations to row reduce the augmented matrix of part (b) to a reduced row-echelon matrix.
- (d) What is the price per pound for each of the three fruits?

- [10] 6. Consider the linear equation $ax + by + cz = d$ ($d \neq 0$) (1)
and the associated homogeneous equation $ax + by + cz = 0$ (2).

Let (x_0, y_0, z_0) and (x_1, y_1, z_1) be two solutions to equation (1) and let (x_2, y_2, z_2) be a solution to equation (2).

- (a) Show that $(x_0 - x_1, y_0 - y_1, z_0 - z_1)$ is a solution to equation (2).
- (b) Show that $(x_0 + x_2, y_0 + y_2, z_0 + z_2)$ is a solution to equation (1).
- (c) Let k be any real number. Show that (kx_2, ky_2, kz_2) is a solution to equation (2).