MATH 1300 ASSIGNMENT PROBLEMS (UNIT 4)

[15]   1. Consider the following matrices:

$$A = \begin{bmatrix} -4 & 1 & 1 \\ 2 & 0 & -2 \end{bmatrix} \quad B = \begin{bmatrix} 1 & -4 & 0 \\ -2 & 0 & 2 \\ -1 & -1 & 1 \end{bmatrix} \quad C = \begin{bmatrix} -1 & 4 \\ 3 & 2 \\ -3 & -2 \end{bmatrix}$$

$$D = \begin{bmatrix} 1 & -2 \\ -2 & 4 \\ 2 & -3 \end{bmatrix} \quad E = \begin{bmatrix} 2 & -3 & 2 \\ -1 & -1 & 2 \end{bmatrix}$$

Calculate, if possible, the following:

(a) $2E - 3A$ (b) $AB + EB$ (c) $E^T + D$

(d) $(B - I_3)C$ (e) $(AB)^2$

[10]   2.(a) Which of the following matrices are elementary matrices?

(i) $\begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (ii) $\begin{bmatrix} 3 & 0 & 0 \\ 0 & 2 & 0 \\ 0 & 0 & 1 \end{bmatrix}$ (iii) $\begin{bmatrix} 0 & 0 & 1 \\ 0 & 1 & 0 \\ 1 & 0 & 0 \end{bmatrix}$

(b) Let A = $\begin{bmatrix} 1 & 4 & 3 \\ 2 & 1 & 5 \\ 3 & 2 & 4 \end{bmatrix}$. Find an elementary matrix E such that EA = B if

(i) B = $\begin{bmatrix} 1 & 4 & 3 \\ 4 & 9 & 11 \\ 3 & 2 & 4 \end{bmatrix}$ (ii) B = $\begin{bmatrix} 3 & 2 & 4 \\ 2 & 1 & 5 \\ 1 & 4 & 3 \end{bmatrix}$ (iii) B = $\begin{bmatrix} 1 & 4 & 3 \\ 2 & 1 & 5 \\ 9 & 6 & 12 \end{bmatrix}$

(c) Let A = $\begin{bmatrix} 2 & 1 \\ 1 & 0 \end{bmatrix}$. Find two elementary matrices E_1 and E_2 such that $E_2 E_1 A = I$.

[10]   3. Consider the matrix

$$A = \begin{bmatrix} 1 & 2 & 3 & 4 \\ 1 & 3 & 3 & 4 \\ 2 & 4 & 7 & 8 \\ 2 & 4 & 6 & 9 \end{bmatrix}.$$

Find the inverse matrix A^{-1}.
Show all your work and verify that your answer is in fact correct.

[10]   4. Consider the matrix $A = \begin{bmatrix} 2 & 1 & 4 \\ 3 & 2 & 5 \\ 0 & -1 & 1 \end{bmatrix}.$

   (a) Find the inverse matrix A^{-1}. Show all your work and verify that your answer is correct.
   (b) Use (a) to solve the system of linear equations
$$2x + y + 4z = 5$$
$$3x + 2y + 5z = 3$$
$$-y + z = 8$$

[5]   5. The state of Onitoba has two types of schools, public schools and private schools. Past experience shows that 90% of the students attending a public school this year will attend a public school next year while 10% will go to a private school next year. Similarly, 80% of those attending a private school this year will attend a private school next year while 20% will go to a public school next year.

   (a) Construct the transition matrix showing the school attendance projections.

   (b) If 70% of the students attend public schools and 30% attend private schools this year, what is the expected distribution for school attendance next year?

   (c) What will be the expected long term steady-state distribution of the students with respect to public and private schools?

[10] 6. The Rent-A-Wreck car rental agency in the city of Zim has 3 outlets, one at the Airport, one in the **B**usiness section of town and the third near the **C**ommunity college. Vehicles rented at one location may be returned at any one of the three locations. In order to run the business more efficiently, the rental agency hires an accounting firm to study the movements of the rental cars. The accounting firm determines that after one week 60% of the vehicles rented at **A** will be returned to **A**, 30% will be returned to **B** and 10% will be returned to **C**. For the vehicles rented at **B**, after one week, 70% will be returned to **B**, 20% will be returned to **A** and 10% will be returned to **C**. Of the vehicles rented at **C**, after one week, 50% will be returned to **C**, 30% will be returned to **A** and 20% will be returned to **B**.

(a) Find the transition matrix for the car rentals after one week.

(b) If the initial distribution of the rental cars is 40% at A, 35% at B and 25% at   C, find the distribution of the rental cars one week later.

(c) What is the probable long run distribution of the vehicles?