MATH 1300 ASSIGNMENT PROBLEMS (UNIT 1)
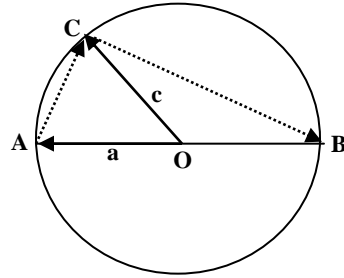
1. AOB is the diameter of a circle with centre at O and C is any other point on the circle. Denote the vector $\overrightarrow{OA}$ by **a** and the vector $\overrightarrow{OC}$ by **c**.

[4] (a) Write the vectors $\overrightarrow{AC}$ and $\overrightarrow{CB}$ in terms of the vectors **a** and **c.**

[6] (b) Use vector methods to show that $\angle ACB$ is a right angle.

2. Let A = (2, 1, −5) and B = (1, −2, 4) be two points in $\mathbf{R}^3$.

[2] (a) Find the components of the vectors $\overrightarrow{AB}$ and $\overrightarrow{BA}$.

[2] (b) Find the coordinates of the point C if $\overrightarrow{AC} = \overrightarrow{CB}$.

[3] (c) The point D = (k, 1, 5) is equidistant from the points A and B. Find the value(s) of k.

[3] (d) Find the coordinates of the point X for which $\overrightarrow{AX} = 2\overrightarrow{AB}$.

3. Let $\mathbf{u} = (2,1,3)$, $\mathbf{v} = (3,2,-2)$ and $\mathbf{w} = (5,-1,4)$ be three vectors in $\mathbf{R}^3$. Find the following.

[2] (a) $2\mathbf{u} - 3\mathbf{v} + \mathbf{w}$

[2] (b) $\mathbf{u} \cdot \mathbf{v}$

[2] (c) $\mathbf{v} \times \mathbf{u}$
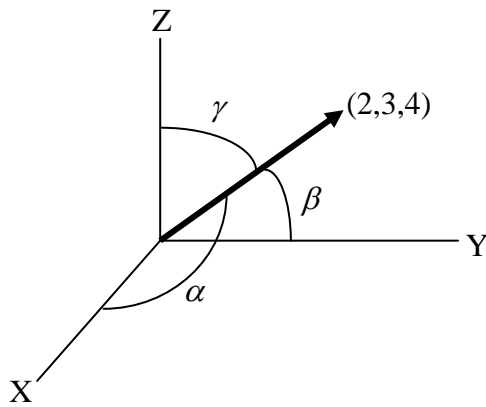
[2] (d) $\text{proj}_\mathbf{v}\mathbf{u}$

[2] (e) sine of the angle between the vectors **u** and **v**.

4. Find value(s) for k so that the angle between $\mathbf{u}=(3,k)$ and $\mathbf{v}=(2,-1)$ is

[3] a) 90^0
[3] b) 0^0
[4] c) 45^0

5. The direction angles of a vector are the three angles α, β and γ that the vector makes with the positive X, Y and Z co-ordinates axes. For the vector (2,3,4) find the following:

[2] a) $\cos\alpha$
[2] b) $\cos\beta$
[2] c) $\cos\gamma$
[4] d) $\cos^2\alpha+\cos^2\beta+\cos^2\gamma$

[10] 6. The four points A(5,0,0), B(0,0,2), C(0,4,0) and D(5,6,–3) form a quadrilateral lying on the plane $4x+5y+10z=20$. Find the area of this quadrilateral.