MATH 1300 ASSIGNMENT PROBLEMS (UNIT 3)

[4] 1. (a) There are 4 possible row-reduced echelon forms of a 2x2 matrix. What are they?

[6] (b) Give an example of two distinct 2x2 nonzero matrices A and B such that AB = 0

[10] 2. Use Gaussian elimination procedure: back substitution to solve the following systems of equations.

a)
$$\begin{aligned} x+y-z &= 0 \\ 2x-y+z &= 6 \\ 3x-y+2z &= 11 \end{aligned}$$

b)
$$\begin{aligned} 2x-y+z &= 0 \\ x-2y+3z &= 3 \\ 4x-5y+7z &= 6 \end{aligned}$$

[10] 3. Use Gauss Jordan elimination procedure to solve the following systems. You must reduce the matrix to its reduced row-echelon form and then write the complete solution to the system.

a)
$$\begin{aligned} x-2y+z &= 3 \\ 2x+y-z &= -8 \\ 3x+2y+3z &= 1 \end{aligned}$$

b)
$$\begin{aligned} w+2x-y+z &= 6 \\ 2w-x+2y-z &= 3 \\ 4w+3x+z &= 15 \\ w-2x+3y-2z &= 8 \end{aligned}$$

[10] 4. The augmented matrix from a system of linear equations has the following reduced row-echelon form.

$$\begin{bmatrix} 1 & 2 & 0 & 1 & 0 & 3 & 0 & 1 \\ 0 & 0 & 1 & 4 & 0 & 5 & 0 & 0 \\ 0 & 0 & 0 & 0 & 1 & 1 & 0 & 2 \\ 0 & 0 & 0 & 0 & 0 & 0 & 1 & 3 \end{bmatrix}$$

(a) How many equations are there in the system?

(b) How many variables are there in the system?

(c) How many parameters are there in the solution set?

(d) Write out the solution set for the system.

[10] 5. Consider the system of linear equations

$$\begin{aligned} x + 2y + z &= 3 \\ 2x + 5y + 3z &= 7 \\ 3x + 7y + 4z &= k \end{aligned}$$

where k is a real number.

(a) Write out the augmented matrix for this system of linear equations.

(b) Use elementary row operations to reduce the augmented matrix to row-echelon form.

(c) For what values of k does the system have solutions? Find all such solutions.

(d) For what value of k does the system have no solution?

[10] 6. Jill, Shirley and Kate went to their local produce store to buy some fruit. Jill bought three pounds of bananas and two pounds of grapes and paid $5.70. Shirley bought two pounds of bananas and two pounds of nectarines and paid $6.20. Kate bought three pounds of grapes and a pound of nectarines and paid $7.80.

(a) Let x = price of a pound of bananas, y = price of a pound of grapes and z = price of a pound of nectarines. Write out 3 linear equations representing the purchases made by Jill, Shirley and Kate.

(b) Write out the augmented matrix for your system of 3 linear equations of part (a).

(c) Use elementary row operations to row reduce the augmented matrix of part (b) to a reduced row-echelon matrix.

(d) What is the price per pound for each of the three fruits?