

MATH 1300 D01/D02 Fall 2015 Assignment 4

SHOW ALL WORK to get full marks. Leave answers as exact answers. For example, leave it as $1/7$ as opposed to 0.142857 .

1. Let $A = \begin{bmatrix} 3 & 2 \\ 4 & -3 \end{bmatrix}$, $B = \begin{bmatrix} -2 & -3 & 4 \\ 0 & 6 & 2 \end{bmatrix}$, $C = \begin{bmatrix} 3 & -3 & -4 \\ 0 & 3 & 5 \\ 4 & 4 & -2 \end{bmatrix}$ evaluate the following operations if possible. If not, explain why.

[5] (a) $AB - 3B$.

[2] (b) $(A - 3)B$.

[2] (c) BA .

[5] (d) $CB^T - (AB)^T$

- [4] 2. In assignment 3, you were asked to find two non-zero matrices A and B such that $AB = 0$. Show that if A and B are square matrices of the same size such that B is an invertible matrix, then A must be a zero matrix. Show every step.

3. For the following system of equations

$$\begin{aligned} 3x + 3y - 2z &= -4 \\ -3x + 2y + 3z &= 1 \\ x - y - z &= -2 \end{aligned}$$

[1] (a) Write the coefficient matrix A .

[6] (b) Use row operations to find A^{-1} .

[3] (c) Use the inverse to solve the system.

4. The matrix $A = \begin{bmatrix} 3 & 0 & 6 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix}$ can be reduced down to the identity in the following three steps

$$A = \begin{bmatrix} 3 & 0 & 6 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 0 & 1 \\ 0 & 1 & 0 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 2 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} \Rightarrow \begin{bmatrix} 1 & 0 & 0 \\ 0 & 1 & 0 \\ 0 & 0 & 1 \end{bmatrix} = I$$

[3] (a) What are the three row operations used in order.

[3] (b) Using part (a), find elementary matrices such that $E_3E_2E_1A = I$.

[2] (c) Write A^{-1} as a product of elementary matrices.

[3] (d) Write A as a product of elementary matrices

5. Let $A = \begin{bmatrix} 1/2 & 3/2 \\ 3/2 & 1/2 \end{bmatrix}$, $D = \begin{bmatrix} 2 & 0 \\ 0 & -1 \end{bmatrix}$ and $P = \begin{bmatrix} 1 & 1 \\ 1 & -1 \end{bmatrix}$.

[5] (a) Compute P^{-1} and verify that $A = PDP^{-1}$.

[2] (b) What is the mistake in the following argument?

" $A = D$ since $A = PDP^{-1} = PP^{-1}D = ID = D$."

[4] (c) Use part (a) to compute A^{2015} .

6. Suppose there are 2 cable companies, Cable A and Cable B. After every month, anybody using Cable A or B people can decide whether to keep their services, switch to the other company or cancel their cable. People who don't have cable can also choose to get cable after each month.

Data shows after each month 80% of cable A customers stay, 10% switch to Cable B and 10% cancel altogether. For cable B customers, 70% of customers stay, 20% switch to Cable A and 10% cancel altogether. For people with no cable 80% of people still have no cable, 15% get Cable B and 5% get Cable A.

[2] (a) Write down the transition matrix. (First column for A, second column for B and third column for no cable)

[8] (b) After a very long time, what percentage will have Cable A, what percentage will have Cable B and what percentage will have no cable.

This assignment is out of 60 points.