MATH 1310 Matrices for Management and Social Sciences

Assignment 4

[10] 1. Let A and B be 4x4 matrices with det(A) = 5 and det (B) = 8. Determine (if possible) the value of the following. If it is not possible to determine the value from the given information, state your answer as **Not Possible**.

(a) $\det(A - B)$

(b) $\det(AB^{-1})^t$

(c) $\det(A^2 B)$

(d) det(adj(A))

(e) det(C) where C is the matrix obtained from A by replacing the first row of A by the sum of the first and second rows of A.

[10] 2. The Zippy Auto Rental Company has two locations, an Airport location and a Business district location. Cars rented at one location may be returned to either location. Past experience shows that after one week 75% of the cars located at the Airport location will still be at the Airport location while 25% of the cars that were at the Airport location will now be located at the Business district location. On the other hand 80% of the cars from the Business district location will still be at this location one week later while 20% of the cars from the business district location will now be at the Airport location.

(a) Construct the transition matrix showing the weekly car return experience for the Zippy Auto Rental Company.

(b) If 40% of the cars are located at the Airport location and 60% are located at the Business district location, determine the distribution of cars one week later.

(c) What will be the expected long term steady-state distribution of the cars at the two rental locations?

[10] 3. Let $A = \begin{bmatrix} k+2 & 2k+4 & k+2 \\ 1 & -2 & 1 \\ 3 & 1 & 0 \end{bmatrix}$. For what values of k is the matrix A invertible?

[20] 4. Use row operations and their determinant properties (not the definition) to find the value of the determinant of the following matrix.

$$\begin{bmatrix} 2 & 4 & 6 & 8 \\ 3 & 6 & 9 & 3 \\ 3 & 1 & 2 & 4 \\ 3 & 1 & 5 & 2 \end{bmatrix}$$

[20**]** 5. Let $M = \begin{bmatrix} 1 & 2 & 3 \\ 4 & 5 & 7 \\ 3 & 8 & 15 \end{bmatrix}$.

(a) Find the matrix adj(M).
(b) Compute the matrix product $M \bullet \text{adj}(M)$.
(c) Use the information from part (b) to determine the value of det(M).
(d) Use the information from parts (a) and (c) to calculate M^{-1}.
(e) Solve the system $MX = N$ where $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $N = \begin{bmatrix} 8 \\ -8 \\ 16 \end{bmatrix}$.

[10] 6. Use Cramer's rule to solve the following system of linear equations.

$$\begin{aligned} 2x + y + z &= 3 \\ 4x - z &= 8 \\ 3x - y + z &= 20 \end{aligned}$$