MATH 1310 Matrices for Management and Social Sciences Assignment 5

[6] 1. Find the vector equation and the parametric equations of the line passing through the points $P(-1, 3, 2)$ and $Q(-4, 3, 1)$.

[9] 2. Let $\mathbf{u} = (4, -3, 1)$, $\mathbf{v} = (2, -2, 3)$, $\mathbf{w} = (5, 3, -1)$. Calculate:
(a) $3\mathbf{u} + \mathbf{v} - 2\mathbf{w}$
(b) $|\mathbf{v} + \mathbf{w}|$
(c) $|-3\mathbf{v}|$

[10] 3. Determine if it is possible to express the vector $\mathbf{v} = (3, -4, 5)$ as a linear combination of the vectors $\mathbf{u}_1 = (-3, 6, -3)$ and $\mathbf{u}_2 = (6, -11, 7)$. If it is possible, write $\mathbf{v}$ as a linear combination of $\mathbf{u}_1$ and $\mathbf{u}_2$.

[10] 4. Find the null space of the given matrix. If the null space is more than the zero vector alone, describe it as the span of a set of vectors.

$$\begin{bmatrix} 4 & -8 & -4 \\ -8 & 15 & 10 \\ 4 & -10 & 1 \end{bmatrix}$$

[10] 5. Determine whether the set $\{(2, 0, 1), (-1, 3, 1), (3, -1, 2\}$ is a spanning set for $\mathbf{R}^3$.

[15] 6. Determine whether or not the set W is a subspace of the given vector space V.
(a) $W = \{(a+b, b) : a, b \text{ real numbers}\}$; $V = \mathbb{R}^2$;
(b) $W = \{(x, 3x, -x) : x \text{ real number}\}$; $V = \mathbb{R}^3$;
(c) $W = \{(1, x, 0) : x \text{ real number}\}$; $V = \mathbb{R}^3$.

[10] 7. Let $\mathbf{u} = (3, -1)$ and $\mathbf{v} = (-2, 4)$. Find all values of k such that $|\mathbf{u} + k\mathbf{v}| = \sqrt{50}$.

[10] 8. Find the value of k for which the following vectors are linearly dependent.
$$\mathbf{u}_1 = (3, 1, k), \quad \mathbf{u}_2 = (-2, 2, 3), \quad \mathbf{u}_3 = (1, 2, -k).$$