MATH 1310: MATRICES FOR MANAGEMENT AND SOCIAL SCIENCES ASSIGNMENT 3

[30] 1. Consider the following matrices:

$$A = \begin{bmatrix} 11 & -2 & 1 \\ 5 & 0 & 4 \end{bmatrix}, B = \begin{bmatrix} 1 & 0 & -3 \\ 1 & 4 & 0 \\ 2 & 2 & -1 \end{bmatrix}, C = \begin{bmatrix} 3 & 1 \\ 4 & -1 \\ 0 & 2 \end{bmatrix}, D = \begin{bmatrix} 1 & 0 \\ 3 & 2 \\ -2 & 10 \end{bmatrix}, E = \begin{bmatrix} 0 & -1 & 5 \\ 1 & 2 & 1 \end{bmatrix}$$

Calculate, if possible, the following:

(a)
$$2A - E$$
 (b) A^2 (c) $BC + BD$ (d) $B + CE$ (e) BD^t (f) $AD + I_2$

$$(g) \ (B+I_3)C \quad (h) \ (AD)^2 \quad (i) \ B(A+E) \quad (j) \ ED^t$$

[10] 2. Find the matrix S such that SM + S = N, where

$$\mathbf{M} = \begin{bmatrix} 0 & 2 & 3 \\ 1 & 2 & 3 \\ 2 & 4 & 6 \end{bmatrix} \quad \text{and} \quad \mathbf{N} = \begin{bmatrix} 2 & 3 & 4 \\ 4 & 2 & 1 \\ 1 & 1 & 3 \end{bmatrix}.$$

- [20] 3. Consider the matrix $A = \begin{bmatrix} 1 & 2 & 0 \\ 2 & 0 & 1 \\ 1 & 2 & 3 \end{bmatrix}$.
 - (a) Find A⁻¹. Show all your work and verify that your answer is correct.
 - (b) Use part (a) to solve the system:

$$x + 2y = 3$$

$$2x + z = 5$$

$$x + 2y + 3z = -2$$

- [20] 4. An economy consists of two sectors, electricity and petroleum. The production of \$1 of electricity requires 40 cents in electricity and 10 cents in petroleum, whereas the production of \$1 of petroleum requires 20 cents in electricity and 30 cents in petroleum. If there is an outside demand for \$3200 of electricity and \$2800 of petroleum, find the necessary production of electricity and petroleum.
- 5. Consider an isolated Pacific island community that has no trade with the outside world. The economy of the island consists of three types of industries: fishing, boat building and fruit gathering. Each industry receives contributions from the other industries. The fishermen keep 2/5 of the fish they catch for themselves and give 3/10 of their production to the boat building industry and 3/10 of their production to the fruit gatherers. The boat builders keep 1/2 of their production for themselves; give 1/4 of their production to the fishermen and 1/4 of their production to the fruit gatherers. The fruit gatherers keep 2/5 of the fruit they gather for themselves, give 2/5 of their production to the fishermen and 1/5 of their production to the boat builders. What is a fair way to assign values of the 3 industries of fishing, boat building and fruit gathering?