MATH 1310 Matrices for Management and Social Sciences

Assignment 4

[10] 1. Let A and B be 4x4 matrices with det(A) = 5 and det (B) = 2. Determine (if possible) the value of the following. If it is not possible to determine the value from the given information, state your answer as **Not Possible**.

(a) $\det(BA^2)$
(b) $\det(BA^{-1})$
(c) $\det(2A + B)$
(d) $\det(\text{adj}(A))$
(e) det(C) where C is the matrix obtained from A by multiplying the second row of A by 3.

[10] 2. The state of Onitoba has two types of schools, public schools and private schools. Past experience shows that 90% of the students attending a public school this year will attend a public school next year while 10% will go to a private school next year. Similarly, 80% of those attending a private school this year will attend a private school next year while 20% will go to a public school next year.

(a) Construct the transition matrix showing the school attendance projections.

(b) If 70% of the students attend public schools and 30% attend private schools this year, what is the expected distribution for school attendance next year?

(c) What will be the expected long term steady-state distribution of the students with respect to public and private schools?

[10] 3. Let $A = \begin{bmatrix} 1 & 2 & k \\ 3 & 6 & k \\ k & 2 & 1 \end{bmatrix}$. For what values of k is the matrix A non-invertible?

[20] 4. Use row operations and their determinant properties (not the definition) to find the value of the determinant of the following matrix.

$$\begin{bmatrix} 2 & 4 & 6 & 8 \\ 1 & 2 & 3 & 1 \\ 3 & 1 & 2 & 4 \\ 6 & 2 & 10 & 4 \end{bmatrix}$$

[20] 5. Let $M = \begin{bmatrix} 3 & 2 & 1 \\ 2 & 6 & 9 \\ 4 & 5 & 7 \end{bmatrix}$.

(a) Find the matrix adj(M).

(b) Compute the matrix product $M \bullet \text{adj}(M)$.

(c) Use the information from part (b) to determine the value of det(M).

(d) Use the information from parts (a) and (c) to calculate M^{-1}.

(e) Solve the system $MX = N$ where $X = \begin{bmatrix} x_1 \\ x_2 \\ x_3 \end{bmatrix}$ and $N = \begin{bmatrix} 42 \\ 21 \\ -42 \end{bmatrix}$.

[10] 6. Use Cramer's rule to solve the following system of linear equations.

$$\begin{aligned} 2x - y + z &= 6 \\ 4x + z &= 13 \\ 3x + y + z &= 14 \end{aligned}$$