MATH 1310 Matrices for Management and Social Sciences Assignment 5

[4] 1. A line passes through the point P (5, 1, 2) and is parallel to the line passing through the points Q (4, 1, 7) and R (3, 8, 4). Find a vector equation and a set of parametric equations for this line.

[6] 2. Let **u** = (4,3,5), **v** = (5,1,4) and **w** = (7, 3, 2) be 3 vectors in $\mathbb{R}^3$. Find the following.

(a) $2\mathbf{u} - 3\mathbf{v} + \mathbf{w}$
(b) A vector **s** such that $2\mathbf{u} + \mathbf{s} = \mathbf{v} - \mathbf{w}$
(c) $|\mathbf{u} - \mathbf{v}|$

[10] 3. Let $A = \begin{bmatrix} 1 & 1 & -2 \\ -1 & 1 & 1 \end{bmatrix}$. Which of the following vectors belongs to the null space of A?

$\mathbf{u_1} = (1,2,3)$, $\mathbf{u_2} = (3,1,2)$, $\mathbf{u_3} = (0,0,0)$, $\mathbf{u_4} = (0,0)$, $\mathbf{u_5} = (6,2,4)$

[5] 4. Find the null space of the given matrix. If the null space is more than the zero vector alone, describe it as the span of a set of vectors.

$$A = \begin{bmatrix} 1 & 1 & 1 & 1 \\ 1 & 2 & 1 & 2 \\ 2 & 3 & 2 & 3 \\ 3 & 5 & 3 & 5 \end{bmatrix}$$

[12] 5. Determine whether or not the set *W* is a subspace of the given vector space *V*.

(a) $W = \{(a+b, 2b) : a, b \text{ real numbers}\}$; $V = \mathbb{R}^2$;
(b) $W = \{(x, x, 2x) : x \text{ real number}\}$; $V = \mathbb{R}^3$;
(c) $W = \{(x, x, 1) : x \text{ real number}\}$; $V = \mathbb{R}^3$.

[5] 6. Let **u** = (3, 4, 5). Find all real numbers *k* such that $|k\mathbf{u}| = 5$.

[6] 7. Find the value of *k* for which the following vectors are linearly dependent.

$$\mathbf{u_1} = (1, 3, k+3), \quad \mathbf{u_2} = (-2, 4, -3), \quad \mathbf{u_3} = (3, -5, k-4).$$

[12] 8. Determine if it is possible to express the vector $\mathbf{v}$ as a linear combination of the vectors $\mathbf{u_1}$ and $\mathbf{u_2}$. If it is possible, write $\mathbf{v}$ as a linear combination of $\mathbf{u_1}$ and $\mathbf{u_2}$.

(a) $\mathbf{u_1} = (-3, 6, 3), \quad \mathbf{u_2} = (6, -11, 7), \quad \mathbf{v} = (3, -4, 5)$

(b) $\mathbf{u_1} = (-3, 6, 3), \quad \mathbf{u_2} = (6, -11, 7), \quad \mathbf{v} = (3, 3, 2)$

(c) Using the information from parts (a) and (b) above, find a basis for $\mathbb{R}^3$ that contains both of the vectors $\mathbf{u_1}$ and $\mathbf{u_2}$.