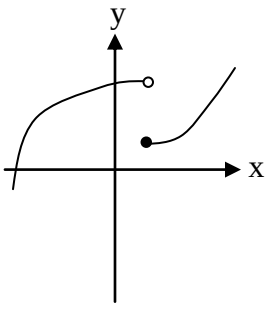
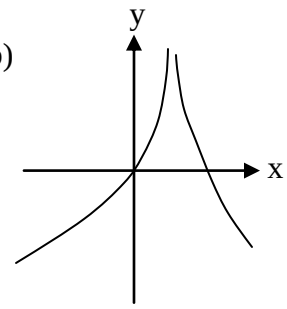
Assignment 2 MATH 1500

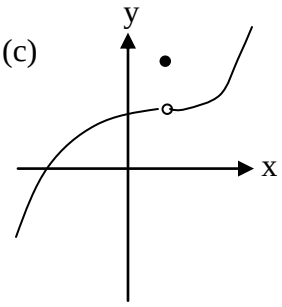
(Follows Unit 7 in the manual)

Values

[6] 1. Each of the following graphs contains a discontinuity. Determine whether the discontinuity is (i) a removable discontinuity (ii) a jump discontinuity or (iii) an infinite discontinuity [cf. Section 2.5]

(a) 

(b) 

(c) 

[6] 2. Each of the following functions contains a discontinuity at $x = 2$. Determine whether the discontinuity at $x = 2$ is (i) a removable discontinuity (ii) a jump discontinuity or (iii) an infinite discontinuity [cf. Section 2.5]

(a) $f(x) = x - 2$ (b) $f(x) = \dfrac{x^2 + 4}{x - 2}$ (c) $f(x) = \dfrac{x^2 - 4}{x - 2}$

[5] 3. Show that the function $f(x) = x^3 - 15x + 1 = 0$ has three solutions in the interval [–4, 4].
[cf. Section 2.5]

[4] 4. Consider the function $f(x) = \begin{cases} x^2 & \text{if } x \le 2 \\ k - x^2 & \text{if } x > 2 \end{cases}$. For what value of k will this function be continuous at $x = 2$? [cf. Section 2.5]

[8] 5. Evaluate the following limits.

(a) $\lim\limits_{x \to \infty} \dfrac{3x^3 - 5x^2 + 7}{8 + 2x - 5x^3}$ (b) $\lim\limits_{x \to -\infty} \left(\sqrt{x^2 + x} - x \right)$

[9] 6. Find horizontal and vertical asymptotes for the following function.

$$f(x) = \frac{x^3 + 1}{x^3 + x}$$

[6] 7. Find an equation of the tangent line to the curve $y = x^3 - 5x + 1$ at the point (1, –3). [cf. Sections 2.7 and 2.8]

[6] 8. Use the definition of the derivative to find the derivative of $f(x) = \frac{2 - x}{2 + x}$. [cf. Sections 2.8 and 2.9]

[15] 9. Find the derivatives of the following functions. Do **not** use the definition of the derivative. You need not simplify your answers.

(a) $f(x) = 5x^{2/3} - 3x^4 + \cos x + e^{-x} + \pi^2$

(b) $f(x) = (\tan x - \ln x)(x^{1/3} + \sin x)$

(c) $f(x) = \frac{3\sqrt{x} + 5\cos x}{2\ln x - 4\sin x}$

[5] 10. Evaluate $\lim_{x \to 0} \frac{\sin(2x)}{\tan x}$. [cf. Section 3.4]

Total = 70