

Total Marks: 60

Due Date: **July 3, 2015.**

SHOW ALL WORK to get full marks. This assignment covers sections 4.1, 4.2, 4.3, 4.5.

1. Find the critical numbers of the function.

(a) (3 points) $f(x) = 2x^3 - 3x^2 - 36x$.

(b) (3 points) $g(t) = |3t - 4|$

2. (5 points) Find the absolute maximum and absolute minimum values of f on the given interval. Justify your answer.

$$f(x) = (x^2 - 4)^3, \quad [-1, 3].$$

3. (6 points) Verify that the following function satisfies the hypotheses of the Mean Value Theorem on the given interval. Then find all numbers c that satisfy the conclusion of the Mean Value Theorem.

$$f(x) = \frac{x}{x + 2}, \quad [1, 4].$$

4. (11 points) Consider $f(x) = x\sqrt{9 - x^2}$. Note then that

$$f'(x) = \frac{9 - 2x^2}{\sqrt{9 - x^2}} \quad \text{and} \quad f''(x) = \frac{x(2x^2 - 27)}{(9 - x^2)^{3/2}}.$$

Find (a) the domain of the function.

(b) the critical numbers.

(c) the open intervals where the function is increasing.

(d) the open intervals where the function is decreasing.

(e) the x and y coordinates of any local(relative) extrema using the first derivative test.

(f) the x and y coordinates of any local(relative) extrema using the second derivative test.

5. Consider the curve given by the function

$$f(x) = \frac{x^2 - 6x + 9}{x - 2}.$$

Then

$$f'(x) = \frac{x^2 - 4x + 3}{(x - 2)^2} \quad \text{and} \quad f''(x) = \frac{2}{(x - 2)^3}.$$

- (a) (2 points) Determine the domain of $f(x)$ and its x -intercepts.
 - (b) (2 points) Find all vertical asymptote(s) of $f(x)$. (Show all your work).
 - (c) (2 points) Find all critical points of $f(x)$ (that is, all critical numbers, together with their y values).
 - (d) (2 points) Find the open intervals where $f(x)$ is increasing and the open intervals where $f(x)$ is decreasing.
 - (e) (2 points) Find the x and y coordinates of all local maxima and/or local minima.
 - (f) (2 points) Determine the open intervals upon which $f(x)$ is concave up and the open intervals where $f(x)$ is concave down.
 - (g) (1 point) Find the x and y coordinates of all inflection point(s).
 - (h) (3 points) Use the above information to give a neat sketch of the graph $y = f(x)$.
6. Consider the curve given by the function

$$f(x) = \frac{-2x}{x^2 - 9}.$$

Then

$$f'(x) = \frac{2(x^2 + 9)}{(x^2 - 9)^2} \quad \text{and} \quad f''(x) = \frac{-4x(x^2 + 27)}{(x^2 - 9)^3}.$$

- (a) (2 points) Determine the domain of $f(x)$ and its x -intercepts.
- (b) (2 points) Find all vertical asymptote(s) of $f(x)$. (Show all your work).
- (c) (2 points) Find all horizontal asymptote(s) of $f(x)$. (Show all your work).
- (d) (1 point) Find all critical points of $f(x)$ (that is, all critical numbers, together with their y values).
- (e) (2 points) Find the open intervals where $f(x)$ is increasing and the open intervals where $f(x)$ is decreasing.
- (f) (1 point) Find the x and y coordinates of all local maxima and/or local minima.
- (g) (2 points) Determine the open intervals upon which $f(x)$ is concave up and the open intervals where $f(x)$ is concave down.
- (h) (1 point) Find the x and y coordinates of all inflection point(s).
- (i) (3 points) Use the above information to give a neat sketch of the graph $y = f(x)$.