MATH 1500 D01/D02 Fall 2015
Assignment 2

SHOW ALL WORK to get full marks. Leave answers as exact answers. For example, leave it as 1/7 as opposed to 0.142857. Word problems should have sentence answers with units. This assignment covers sections from 2.5–2.8 and 3.1–3.2.

Use the definition of derivative ONLY for Q5. Do not use the definition of derivative in any other questions.

[7] 1. Use limits to calculate a and b such that the following function is continuous for all real numbers x.

$$f(x) = \begin{cases} \sin\left(\dfrac{\pi}{2}x\right) + ax^2 + b & x < 2 \\ 3 & x = 2 \\ e^{(x-2)} + a + b + \ln(x-1) & x > 2 \end{cases}$$

[4] 2. Show that $\sin\left(\dfrac{\pi x}{2}\right) = 6x - 4$ has a solution on the interval $(0,1)$. Justify your answer.

3. Evaluate the following limits:

[4] (a) $\displaystyle\lim_{x\to-\infty}\left(e^x + \dfrac{1+x^2}{9x^2-7}\right)$.

[4] (b) $\displaystyle\lim_{x\to-\infty}\dfrac{\sqrt{x^6-x+1}}{2x^3-1}$.

[5] (c) $\displaystyle\lim_{x\to\infty}\left(2x - \sqrt{x^2+1}\right)$.

[6] 4. Use limits to determine all horizontal and vertical asymptotes for the function

$$f(x) = \dfrac{\sqrt{x^2+1}}{-x+1}.$$

[7] 5. Use the definition of the derivative to calculate the derivative of the function $f(x) = \dfrac{1}{x^2+x+1}$.

6. Calculate the derivative of the following functions. Do not simplify your answers.

[3] (a) $f(x) = x^{2015\pi} + \dfrac{7}{x^{10}} + \dfrac{1}{\sqrt[3]{x^4}}$

[4] (b) $f(x) = (2x + e^x)x$

7. Lilly shoots a free throw at her basketball game, its height (in feet) is given by the formula $h(t) = -t^2 + 8t$ where t is in seconds.

[1] (a) Find the velocity at any time t seconds.

[3] (b) After how many seconds will the ball stop rising and begin to fall?

[4] (c) After what time, and with what velocity will ball hit the ground?

[4] 8. Let $h(x) = \dfrac{2g(x) + 1}{f(x) - 1}$.

Find $h'(5)$ if $f'(5) = 1 = g'(5)$, and $f(5) = -1 = g(5)$.

[4] 9. Calculate an equation of the tangent line to $f(x) = \dfrac{x^2 - x - 1}{e^x + 1}$ when $x = 0$.

This assignment is out of 60 points.